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# HOW SHORT IS A SEMI-INFINITE LADDER? AN EXPERIMENTAL APPROACH

**L. CSILLAG[1], A. TUNYAGI[1], ZS. LÁZÁR[1] and A. SIMON[1*]**

**ABSTRACT.** This paper presents the challenging problem of solving of a semi-infinite resistive ladder and highlights some traps and tricks of the subject. The topic is approached from and experimental point of view by solving it using computation, simulation and measurements. All the work was done with the hope that it can be an aid for Physics or Electrical Engineering teachers, students eager to learn and understand more, and to be usefully incorporated into the educational process of talented pupils and students.

***Keywords:*** *semi-infinite resistive ladder, equivalent resistance, experimental approach, analytical solution*

## INTRODUCTION

Electrical circuits and networks are important components of high-school, college and university teaching curricula. At high-school level, the teaching process is based only on Ohm's and Kirchhoff's laws with simple calculus, but later, at undergraduate level, this knowledge is being completed by several new laws and network theorems (Thevenin, Norton, Millman, etc.) and some advanced Mathematics.

At introductory levels of teaching it is customary to illustrate the above-mentioned laws and theorems with the general problem of calculating the equivalent resistance (or impedance) of some, good looking, not very complicated series, parallel or mixed arrangements and combinations of elements. More complicated arrangements (cube, hexagon, flower, ladder, square grid, toroid, polyhedron, fractals, etc.) of a larger number of elements, or even an infinity of them, have several solving

[1] *Babeș-Bolyai University, Faculty of Physics, 1 M. Kogălniceanu street, 400084 Cluj-Napoca, Romania*
* *Corresponding author: alpar.simon@phys.ubbcluj.ro*

traps or tricks, but leading towards some elegant solutions [1-21]. Moreover, the experimental approach to this problem is rather neglected in the scientific literature [4, 5, 22, 23].

Including infinity in those networks, at least at the first sight, tends to complicate the solving of the problem, and making that type of problem being an excellent example and support of the general idea that only classical, fundamental knowledge is not sufficient to cope with such problems, open mindedness and both flexibility and creativity in thinking is necessary.

The problem of infinite resistance (or equivalent) is a very interesting and challenging topic from both scientific and pedagogic point of view. The scientific or practical applications may include transmission lines and networks, filters, digital to analog conversion, geophysical prospection, automatized error testing etc. The theoretical and pedagogical aspects are dealing with some important topics of Physics, Mathematics or Electrical Engineering teaching such as random walk, discrete variable Fourier transforms, lattice Green's function, golden ratio, Fibonacci sequence, computer-based simulation and programming, etc.

University students could land in difficulty when try to solve an infinite network. In fact, this later ascertainment, made on own students, was the starting point of this paper: a short (15 minute) test was given to $2^{nd}$ year Physics students at the end of a complete, one semester long Electricity and Magnetism Course, the task was to find the equivalent resistance of an infinite resistive ladder. The results of the test are supporting the general idea that only classical, fundamental knowledge is not sufficient to cope with such problems, open mindedness and both flexibility and creativity in thinking is necessary: 14.8 % of the students approached the problem via network theorems, but getting into an impasse after analyzing the first two loops, 40.7 % of them approached the problem via mathematical induction, from one unit towards the $n^{th}$ one, but getting into a dead end after analyzing just two loops, 26 % have written the expressions of series and parallel resistances completely wrong after the first loop, finding out no solution for the network and finally 18.5 % of them solved the problem correctly, but all of them were former participants to Physics problem solving contest during high-school.

As we stated before, the study of infinite networks goes beyond fundamental knowledge, requiring creative and flexible thinking. This paper is trying to deal with the teaching challenges of a semi-infinite resistive ladder and approaching to it from both analytical and experimental point of views. We have tried to solve it using computation, simulation and measurements, in all cases highlighting the traps and tricks of the subject, emphasizing on how the analytical and experimental approaches, together with simulation sustain and complete each other. All the work was done with the hope that it can be usefully incorporated into the educational process of talented pupils and students.

## THE ANALYTICAL APPROACH TO THE RESISTIVE LADDER
## A. THE GENERAL CASE ($R_1 \neq R_2$)

The ladder circuit, or attenuation chain, is a simpler particular case of the two-dimensional infinite network of resistors. It is semi-infinite because it has a pair of starting connection point (A and B) and extends towards infinity only in one direction. It is built by successive adding of units of loops of series connected resistors $R_1$ and $R_2$, the next unit being always connected in parallel with resistor $R_2$ from the previous unit (Fig. 1). The eternal question is to find the equivalent resistance $R_{eq}^{(n)}$ of such a ladder [24, 25]

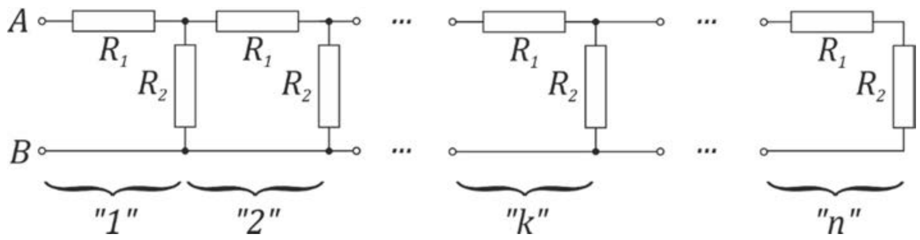



**Fig. 1** Semi-infinite resistive ladder

Usually, the problems involving infinity may intimidate solvers and classical approaches may not work, but in the same time, such a problem will be very useful as an educational tool towards developing a scientific open mindedness, flexibility and creativity in thinking.

Finding out the equivalent resistance of such an infinite circuit is a well-known classical problem, with an elegant solution and several, both theoretical or experimental solving approaches, but the nodal voltages, branch currents, attenuation for the same type of network are less more discussed in the literature.

In spite of the infinite number of resistors the series connected resistors tend to increase, the parallel connected ones tend to decrease the equivalent resistance. Therefore, the expected equivalent resistance will not be infinite due to the mutual compensation of the effects induced by both $R_1$ and $R_2$, and the nodal voltages and branch currents are expected to be almost zero at far end of the ladder.

In the case of infinities, the solver is tempted to try mathematical induction: start with two repeating blocks, calculate the equivalent resistance, then increase the number of blocks by one, calculate again, and so on … until some recursive formula is found, but it is a hard work with persistence and extra attention being crucial during deduction. And the result is not very foreseeable.

As it was also highlighted by Kagan and Wang [26], here comes the first trap of the solving: when increasing the number of repeating blocks, the new pair of $R_1$ and $R_2$ could be added to the left or to the right from the existing circuit. Adding to the right has an experimental, circuit building technique type of logic, and contrarily, the adding to the left has apparently no justification, being the only alternative to the right side of the ladder. This latter approach may be supported also by the logical thinking that suggests to extend the finite end of the ladder (from points $A$ and $B$ to the left, Fig.1) and not the far-right end, already being at infinity.

Anyway, the trick of solving a semi-infinite ladder is simple and elegant, having nothing related to Physics or Electrical Engineering, but only to logical thinking: if the ladder is semi-infinite, already containing an infinity of resistor pairs, its equivalent resistance is not depending on the number of pair units ($n$) and will not be changed by adding one more unit (or even cutting down one, if the ladder already existed):

$$R_{eq}^{(n+1)} = R_{eq}^{(n)} = R_{eq}^{(n-1)} = R_{eq}^{(\infty)} \tag{1}$$

How to get out from the trap of the “ladder end”? Kagan and Wang [26] suggest that adding one more unit to the right of the existing circuit would violate the assumption that in the rightmost unit $R_1$ and $R_2$ are in series. This assumption needs some correcting amendments: (i) by adding the new pair of $R_1$ and $R_2$ to the right of the ladder already containing $n$ units, we increase this number to $n+1$, and face a parallel connection between the the $n^{\text{th}}$ $R_2$ and the new unit ($R_1$, $R_2$), but in the new last unit (the $n+1^{\text{th}}$ one) $R_1$ and $R_2$ will still be in series (see Fig.2), (ii) when adding a new unit from the right of the ladder, the connection is not made between reference points $A$ and $B$, representing the desired equivalent resistance given by Eq.1, but between points $B$ and $C$ (see Fig. 2) and the presumption of not changing the equivalent resistance $R_{eq}^{(\infty)}$ will not stand anymore!

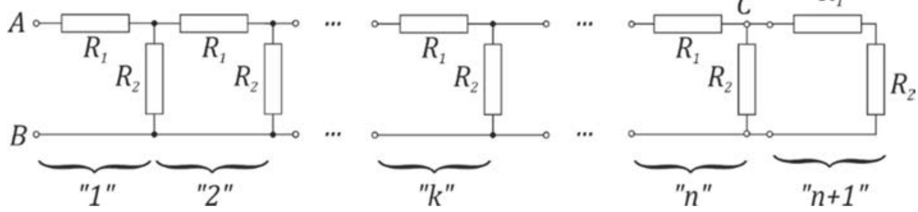
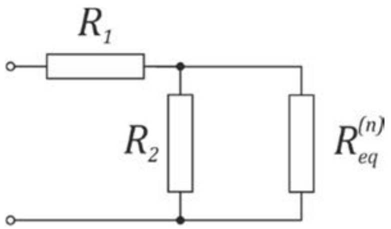



**Fig. 2** Adding an extra unit to the right of the ladder

Let's analyze now the finite end of the ladder. By adding the new pair of $R_1$ and $R_2$ to the left of the ladder already containing $n$ units we will also increase this number to $n+1$, and have to deal with a parallel connection between $R_2$ and the initial ladder, all being in series connection to $R_1$. Thus, the equivalent resistance in this case will be (see Fig. 3, Eq.1 and Eq.2):




$$R_{eq}^{(n+1)} = R_1 + \frac{R_2 \cdot R_{eq}^{(n)}}{R_2 + R_{eq}^{(n)}} = R_{eq}^{(n)} \tag{2}$$

**Fig. 3** Adding an extra unit to the left of the ladder

If somebody will approach the semi-infinite ladder equivalent resistance by computer coding, the above listed equation will be useful for recursion.

When trying to solve Eq.2 for $R_{eq}^{(n)}$ , one can find:

$$\left(R_{eq}^{(n)}\right)^2 - R_1 \cdot R_{eq}^{(n)} - R_1 \cdot R_2 = 0 \tag{3}$$

with

$$R_{eq}^{(n=\infty)} = \frac{R_1}{2} \cdot \left[1 + \sqrt{1 + 4 \cdot \left(\frac{R_2}{R_1}\right)}\right] \tag{4}$$

being the only positive root. This relationship is not depending on $n$, being a finite value determined only by the values of $R_1$ and $R_2$, and represent the correct solution to the problem of the equivalent resistance of a semi-infinite ladder.

The nodal voltages ($U_k$) and branch currents ($I_k^s, I_k^p$) are illustrated in Fig. 4:

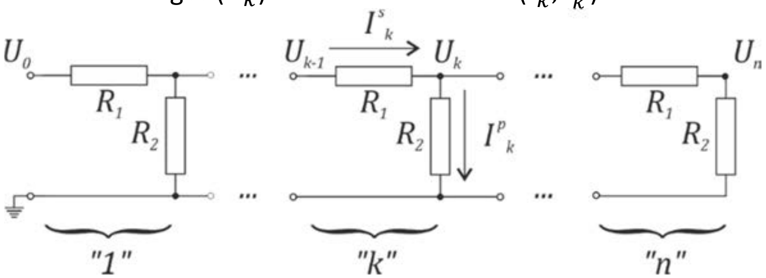



**Fig. 4** Branch currents and nodal voltages along the ladder

By definition, the intensity of the currents in the series and parallel branches will be given by:

$$I_k^s = \frac{U_{k-1} - U_k}{R_1} \tag{5}$$

$$I_k^p = \frac{U_k}{R_2} \tag{6}$$

To find out an analytical expression for the nodal voltage $U_k$ as function of both node position ($k$) and voltage existing at the previous node $U_{k-1}$ or the input voltage $U_0$ is very laborious and will lead towards some very complicated looking polynomial ratios. The same is the situation if derive the transfer function $A$, representing the ratio between the voltage measured at the last node $n$ and the input voltage $U_0$.

**B. The particular case ($\boldsymbol{R_1 = R_2 = R}$)**

One of the most popular and mathematically beautiful particular case of the infinite or semi-infinite ladder is found when both resistors are equal in value because all the electrical characteristics of the ladder will be expressed by means of the Fibonacci numbers ($F_n$) or the golden ratio ($\varphi$ = 1.6180339887) [27-29]:

$$F_n = \begin{cases} 0\,, n = 0 \\ 1\,, n = 1 \\ F_{n-1} + F_{n-2}\,, n \geq 2 \end{cases} \quad \text{with} \quad \lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \varphi$$

In this case, the previously mentioned electrical parameters for the semi-infinite ladder, as function of the number of the ($R_1$, $R_2$) units, will become [30, 31]:

$$R_{eq}^{(n)} = R \cdot \frac{F_{2n+1}}{F_n} \tag{7}$$

$$U_k = \frac{F_{2(n-k)+1}}{F_{2n+1}} U_0 = \frac{F_{2(n-k)+1}}{F_{2(n-k)+3}} U_{k-1} \tag{8}$$

$$I_k^s = F_{2(n-k)+2} \cdot I_n^s \tag{5'}$$

$$I_k^p = F_{2(n-k)+1} \cdot I_n^p \tag{6'}$$

$$A = \frac{U_n}{U_0} = \frac{1}{F_{2n+1}} \tag{9}$$

**C. A real case (1 $k\Omega$, ± 1 % tolerance resistors)**

None the less, the results presented above are all "good looking" and leading towards predictable parameters, no one is tempted to move towards ladder design and experiment. Why? Because any experimental attempt regarding infinite or semi-infinite ladders seems to be overshadowed by two obstacles: (i) no one can build an infinite or semi-infinite circuit, even if uses tens of thousands or billions of resistors , (ii) cannot find resistors with exactly the same value except for those with very high precision (they are very expensive, a resistor with ± 0.1 % tolerance being significantly more expensive than one with a 1 % tolerance).

As a good compromise (acceptable tolerance for low price), let us consider, for simplicity in calculus, too … a batch of 1 kΩ, 0.5 W, 1 % tolerance resistors.

The worst-case scenario for design is when all resistors have resistances larger (or smaller) with exactly 1 %. If we consider the equivalent resistance of the semi-infinite ladder, given by Eq.4, to be function of $R_1$ and $R_2$, the uncertainty in $R_{eq}^{(n=\infty)}$, denoted $\Delta R_{eq}^{(\infty)}$, will be according to [32]:

$$\Delta R_{eq}^{(\infty)} = \sqrt{\left(\frac{\partial R_{eq}^{(\infty)}}{\partial R_1}\right)^2 \cdot \Delta {R_1}^2 + \left(\frac{\partial R_{eq}^{(\infty)}}{\partial R_2}\right)^2 \cdot \Delta {R_2}^2} \tag{10}$$

where

$$\frac{\partial R_{eq}^{(\infty)}}{\partial R_1} = \frac{\frac{1}{2} \cdot \sqrt{1 + 4 \cdot \left(\frac{R_2}{R_1}\right)} + \frac{1}{2} + \left(\frac{R_2}{R_1}\right)}{\sqrt{1 + 4 \cdot \left(\frac{R_2}{R_1}\right)}} \tag{11}$$

$$\frac{\partial R_{eq}^{(\infty)}}{\partial R_2} = \frac{1}{\sqrt{1 + 4 \cdot \left(\frac{R_2}{R_1}\right)}} \tag{12}$$

In the above listed relations $R_1 = R_2 = 1$ kΩ, $\Delta R_1 = \Delta R_2 =$0.01 kΩ.

Thus, the maximum uncertainty for the equivalent resistance in our case might be $\pm$ 0.011717 kΩ (11.717 Ω), meaning that any equivalent resistance value in the interval 1606.317 Ω and 1629.751 Ω, around the value given by the golden ratio (1618.034 Ω) , is acceptable.

What happens in a more realistic scenario, where the individual values of the resistors from the batch are spread around the value of 1 kΩ, but in the limit of the 1 % tolerance? We can estimate the level of uncertainty in the limiting net equivalent resistance, and obviously expect a narrower interval than that from the worst-case scenario.

If we have for the $(R_1, R_2)$ units the mean values denoted $\langle R_1 \rangle$ and $\langle R_2 \rangle$ respectively, with small standard deviations $\sigma_1$ and $\sigma_2$, the uncertainty for the equivalent resistance will be given by

$$\sigma = \sqrt{\frac{\sigma_1^2 + \alpha^2 \cdot \sigma_2^2}{1 - \beta^2}} \tag{13}$$

$$\alpha = \left(\frac{R_2}{R_{eq} + R_2}\right)^2 \tag{14}$$

$$\beta = \left(\frac{R_{eq}}{R_{eq} + R_2}\right)^2 \tag{15}$$

This uncertainty will be calculated later in this paper, when discussing the experimental approach to the ladder, using real, commercially available resistors.

## THE EXPERIMENTAL APPROACH TO THE RESISTIVE LADDER

In our opinion, it is not necessary to buy a tremendously large number of very good tolerance value resistors in order to construct and study an almost semi-infinite ladder, because as explained previously, the semi-infinite ladder has a finite equivalent resistance, and therefore, a not so large number of $(R_1, R_2)$ units has to exist from where the equivalent resistance will converge, with an acceptable accuracy, to its value at infinity.

Coding is an excellent tool in order to find out such a value and has no difficulties with very large number of ladder units. We have developed a very simple Python code [33, 34] to find out the number of units $(k)$ for which the equivalent resistance is still larger or equal to the golden ratio [29]. The recursive formula for coding the equivalent resistance was adapted from Eq. 3, and a number of 10,000 units were initially targeted.

As one can see, surprisingly the condition for the equivalent resistance being larger or equal to the golden ratio is valid only for up to 15 units and the relative error becomes only 0.0626 % starting from the 4$^{th}$ unit. Thus, a semi-infinite ladder formed by identical resistors seems to be very short, nor longer than 15 units! Anyway, if trying to construct experimentally the ladder and make some experimental measurements, it would be useful and wise to consider a little bit larger number of units (20 in our case).

Although, according to the analytical and electric network analysis approach, the adding of a unit to the right end of the ladder is not correct and will not lead to the right solution. In our opinion, such an approach makes sense from experimental point of view: when building up a circuit, you start from the connection points (A and B, Fig.1), and add new units.

We have started to test our hypothesis using a digital multimeter (DMM) [35], a soldering breadboard [36] and 40 resistors (1 kΩ, 0.5 W, 1 %) [37]. The soldering type breadboard was preferred in order to avoid disturbing influences due to less perfect contacts. The randomly chosen pairs of $R_1$ and $R_2$ resistors were each measured and then successively mounted and soldered on the breadboard (Fig. 5). The equivalent resistance $R_{eq}^{(n)}$ was permanently monitored with the DMM.

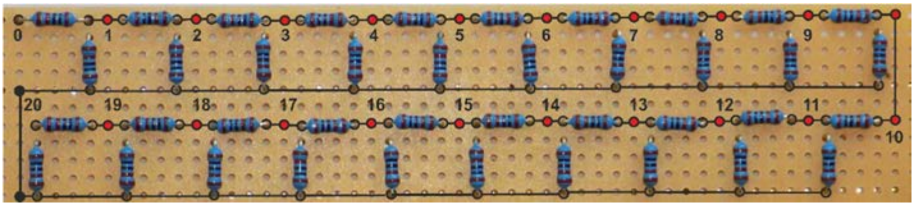



**Fig. 5** A 20 units long resistive ladder on a soldering breadboard
(numbered points are measurement nodes; continuous line is the ground)

In order to have a sort of benchmark, a SPICE-based analog simulation program was used [38]. The results are depicted in Fig. 6.

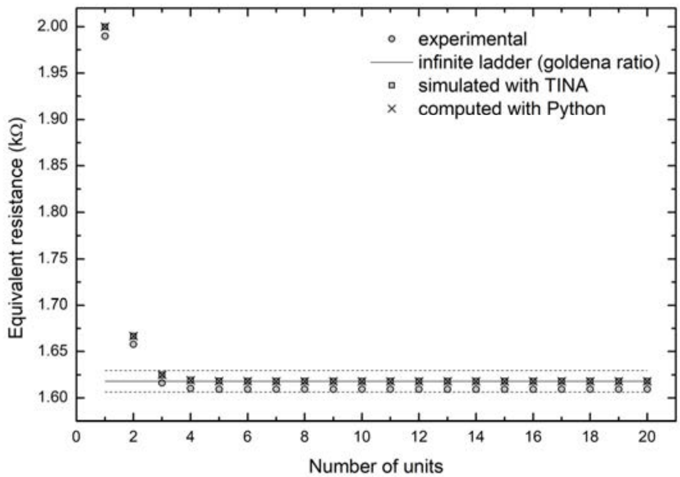



**Fig. 6** Equivalent resistance of the ladder: experiment vs. theory

As one can see, and as it was expected, the convergence of the experimentally measured equivalent resistance is observed starting from the 4th unit (like in the case of coding), the relative error being around 0.48 % and the equivalent resistance being well inside the worst-case scenario interval (dashed lines), therefore our first hypothesis was confirmed. There is a good agreement between data, the measured equivalent resistances being smaller than the computed/simulated ones because the exact, real values of the purchased resistances used in our experiment were all less than 1 kΩ: $\langle R_1 \rangle = 0.99488$ kΩ, $\sigma_1 =$0.000891 kΩ, $\langle R_2 \rangle =$0.99588 kΩ, $\sigma_2 = 0.001298$ kΩ and $\sigma = 0.001031$ kΩ which is significantly less than the 0.011717 kΩ for the worst-case scenario.

Fig. 7 depicts the influence of the adding (or removing) order on the equivalent resistance. Firstly, the units were added to the right end, then they were removed from the right end. Finally, the 20-unit ladder was constructed again from the beginning and then the units were removed from the left end towards the 20th unit.

Fig. 7 validate and strengthen our second hypothesis regarding equivalent resistance: from experimental point of view it does not matter if you add or remove, from the so called “finite” or “infinite” end, the equivalent resistance will have the same dependence on the number of units and there is an excellent agreement between the different approaches.

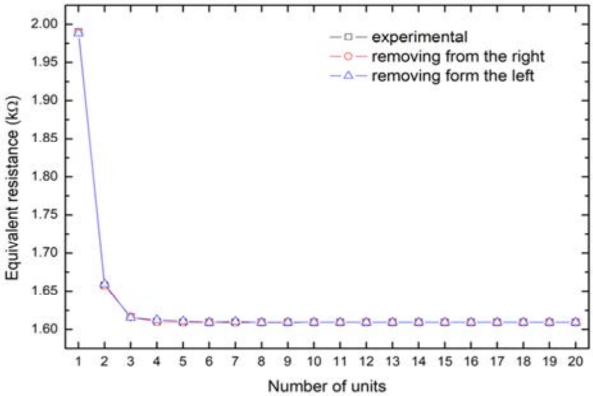



**Fig. 7** Equivalent resistance evolution

Starting from the 14th unit the DMM used for measurements is not capable to reveal any changes in the value of the equivalent resistance, therefore experiment (more precisely the experimental apparatus accuracy) might "shorten" the ladder, probably to 14 units in our case.

According to the literature [4] an exponential decrease is expected for the currents in successive loops. In order to study this dependence, a programable DC power supply was used [39] to provide a steady 10 V input voltage ($U_0$), the nodal voltages being measured with the above mentioned DMM and the currents were calculated using the previously mentioned formulas (Eq.5 and 6). The voltage of each node and the branch currents are presented in Fig. 8, Fig. 9 and Fig. 10, respectively, the inserts representing the same plots, but on log-normal scale.

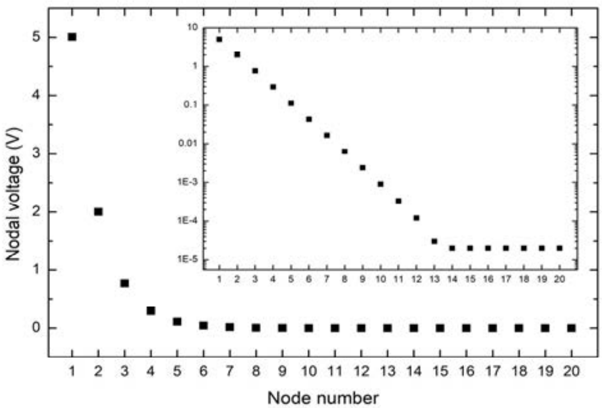



**Fig. 8** Voltages measured in each node

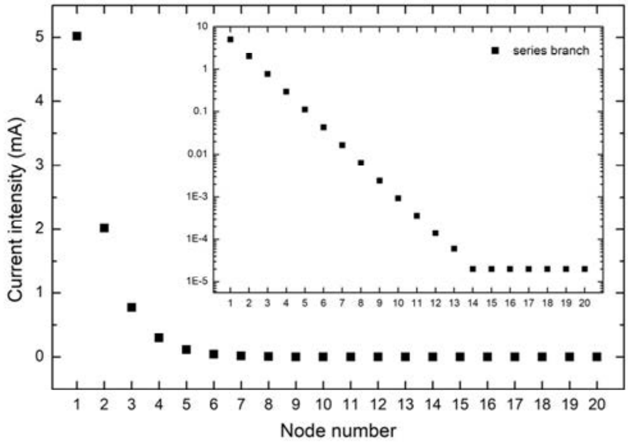



**Fig. 9** Current intensity in series branches

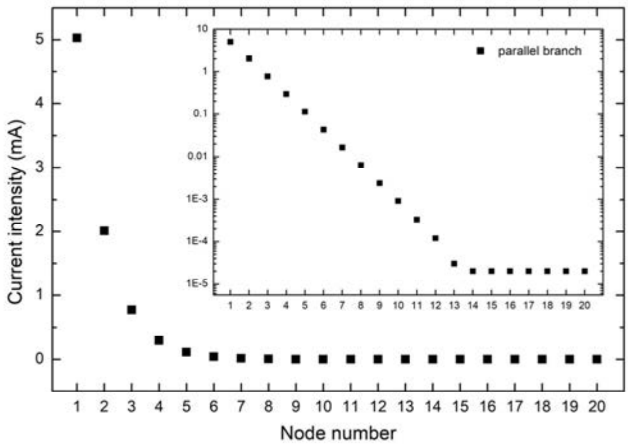



**Fig. 10** Current intensity in parallel branches

As it was suspected by Denardo et al [4], voltage and currents present exponential decrease. Starting from node 14 the voltages are constant because the measurement precision of the DMM is exceeded and therefore are not reliable and appropriate for any scientific conclusions.

For the first 14 nodes, the evolution of the nodal voltages as function of the number of the ($R_1$, $R_2$) units is presented in Fig. 11, meanwhile Fig. 12 depicts the evolution of the relative error (as compared to theoretical value).

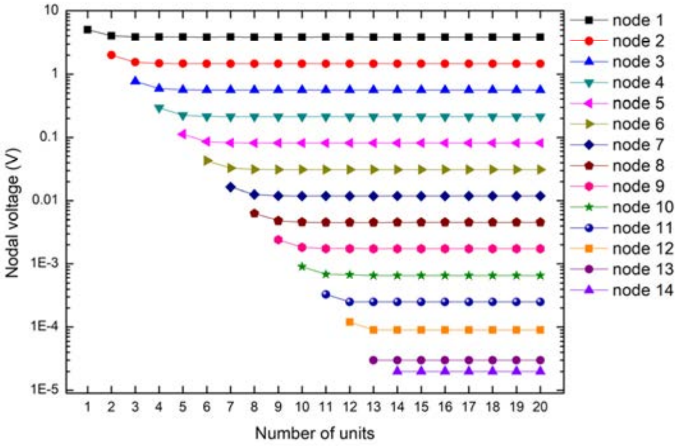



**Fig. 11** Evolution of the nodal voltages as function of the number of units

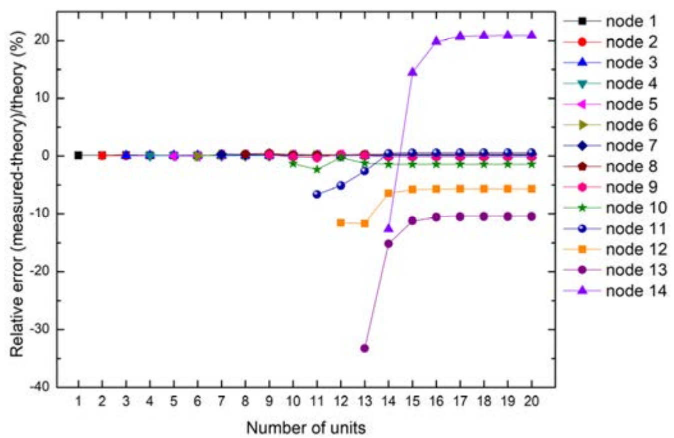
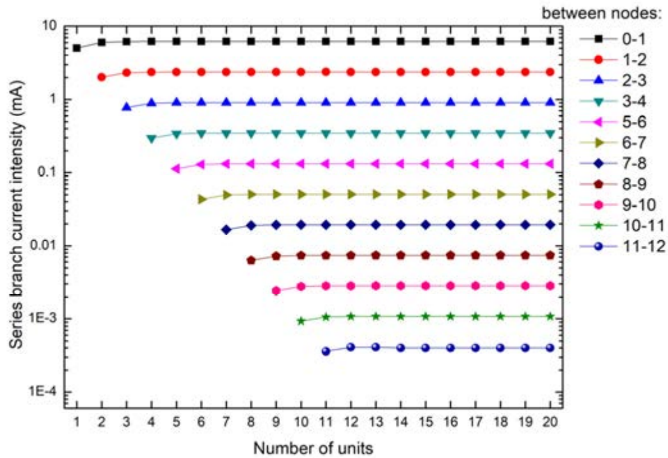
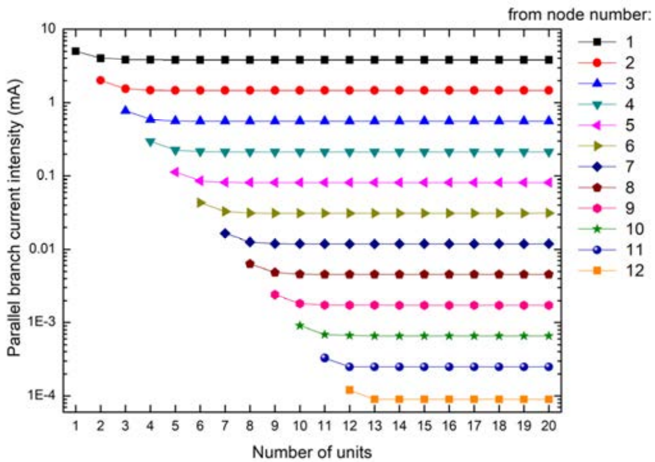



**Fig. 12** Evolution of the relative error as function of the number of units

As one can observe on Fig. 12, the relative error become significant (≥ 5%) starting from node 12 and unacceptable from here onwards. The branch currents deduced from the nodal voltages, for the first 12 nodes are presented in Fig. 13 (a) and (b).




(a)




(b)

**Fig. 13** Evolution of series (a) and parallel (b) branch currents as function of the number of units

As demonstrated above, in both resistance or voltage measurements the experimental apparatus accuracy will "shorten" the ladder, down to 12 units in our case.

## CONCLUSION

A semi-infinite resistive ladder was approached form both analytical and experimental (coding, simulation, measurements) point of views. It was demonstrated that in order to study the behavior of a such ladder it is not necessary to use a very large number of resistors: coding proved that the system tends to converge rapidly (only after about 15 units) towards a constant, finite value. This length of the ladder might be shortened by the accuracy of the experimental apparatus (digital multimeter) and experimental error. The results presented in this paper proved that from experimental measurement point of approach is doesn’t matter which end of the ladder will be extended, the same steady, finite value will be reached.

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