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SIMPLE METHOD FOR DIGITAL DATA RECORDING OF DAMPED OSCILLATIONS OF MECHANICAL PENDULUM

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ABSTRACT. A simple method for digital data acquisition of low frequency damped oscillations of mechanical pendulum is proposed. The method use very simple equipment available in every didactic laboratory. It is intended for didactic purpose, but the method can be used for scientific investigations also.

Keywords: *digital data recording, low frequency oscillations, mechanical pendulum*

INTRODUCTION

The digital acquisition of data is today a very common stage of every scientific experiment, being performed with equipments implemented in every modern system for experimental investigation. This situation facilitates a rapid and reliable acquisition of data without special skills requested to the

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user, [1-4]. However the simplicity of the use is paid some time by the incomplete understanding of the physics and mathematics implemented into the sophisticate procedure of data acquisition. This knowledge lacking leads to the impossibility to adjust or to improve the parameters of the acquisition. Creating an opportunity to operate inside the protocol of data recording represents the aim of this work. We propose a simple method for digital data acquisition of the damped oscillations of the mechanical pendulum using only an ordinary computer mouse, a common computer and the software Processing, available free on the internet. The method is intended for application in the didactic experiments, being accessible to a large category of population. Despite the simplicity the collected data are very accurate and can be used also for scientific investigations.

EXPERIMENTAL

The main piece of the experimental set-up is a home-made mechanical pendulum attached to a common CD which rotates in front of an optical mouse (Fig. 1). The pendulum is made from a thin and light rod of aluminium or steel of length *l=33 cm* and neglecting mass and a small heavy body of mass *m=0.73 g*. The length and the mass of pendulum can be set according to the necessities of the experimentations. The other end of the rod is attached to the CD, which can rotate around a fixed axle passing through its mass center. It is suitable to use a bearing in order to reduce at minimum the friction and to ensure a stable plan of rotation. We used for this purpose an old computer hard disc drive, (HDD). Only the mechanical bearing is used, the other parts, like the lecture head and the positioning system of the head, are eliminated. This system ensures a very precise plan of oscillations and very small friction. In front of the CD, on the vertical plane of the pendulum, we placed an optical mouse, at *0.5-1 mm* distance from the CD, avoiding any friction with the CD. The distance between the axle of the CD and the optical sensor of the mouse is *b=5 cm*. When the pendulum oscillates, the CD move in front of the mouse that is equivalent with a motion of the mouse along a segment of circle. The position of the mouse is transformed by its processor on *X* and *Y* coordinates transmitted to the computer. We process these coordinates in order to display and calculate the parameters of the oscillation.

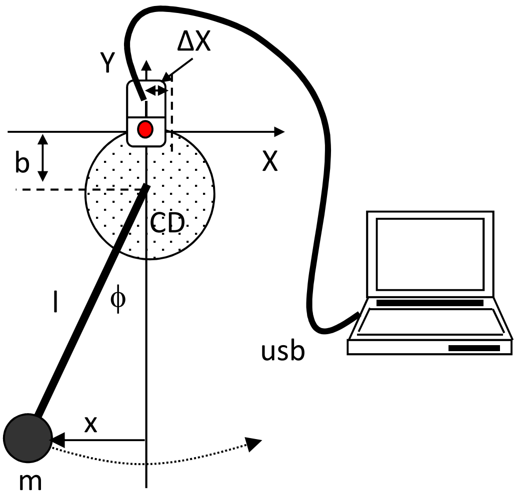

Fig. 1. Schematic representation of the experimental setup.

RESULTS AND DISCUSSION

Digital data acquisition from a moving object supposes the use of adequate motion sensor. Usually mechanical or optical encoders dedicated to each application are used. The accuracy of measurements depends on the resolution of the sensor. The accuracy is defined by the rate of digital sampling per unit of length. More the resolution is high more the sensors are complex and expensive. A simple solution is the use of an optical mouse. Usually such device has a resolution of hundreds of dpi/mm, enough to ensure tens of readings for displacements of the order of few millimeters. Such characteristics are adequate to our situation. Small oscillations of the pendulum, of angles less than 4 degrees, determine a displacement of the CD in front of the optical sensor of the mouse $\Delta X = b \cdot sin\phi \cong b \cdot \phi$ of the order of few millimeters. This displacement is equivalent with a displacement of the optical mouse of the order of tens of digital points along the OX axis. In the ordinary computer the data read by the mouse are transferred directly to the processor, being not accessible for external use. However we can read and display on the PC the coordinates of the mouse using Processing software. To do this we must download the this software from official site and install it on the computer, [5]. Then we upload the following code:

```
//The code

import processing.serial.*;
int start;
int count;
int prevX=200;
int X=100;
void setup() {
size(600, 400);
background(255);
smooth();
}

void draw() {
 int X =mouseX;
 if(X>prevX) {
 start = millis();
 count++;
```

```
  stroke(0);
  ellipse(X,count,5,5);
prevX=X;
}
if(X<prevX) {
start = millis();
count++;
stroke(0);
ellipse(X,count,5,5);
println(X,",",count,";");
prevX=X;
}
}

//End of the code.
```

The code starts automatically if the pendulum is moving. It is enough to displace the oscillating body from its equilibrium position and leave it to oscillate freely. Processing read the coordinate X of the mouse and starts automatically the time counter. The instruction for reading the X coordinate is:

```
int X =mouseX;
```

The current position of the mouse is compared with the previous value.

```
if(X>prevX) {
start = millis();
count++;
```

If the new value is greater than the previous, the counter is started. The variable *count* measures the time of data sampling. In our code the sampling is performed to each millisecond. The counter is started also if the current coordinate is smaller than the previous.

```
if(X<prevX) {
start = millis();
count++;
```

If one of these conditions are fulfilled, the computer receive the instruction to draw a small circle with radius 5 points, centered at (count, X).

```
stroke(0);
ellipse(X,count,5,5);
```

The amplitude of oscillations, designed by the variable *X*, and the time, designed by the variable *count*, are graphically represented in the working windows of Processing. In the same time the data are written in two columns of the Processing working windows, (Fig. 2).

```
println(X,",",count,";");
```

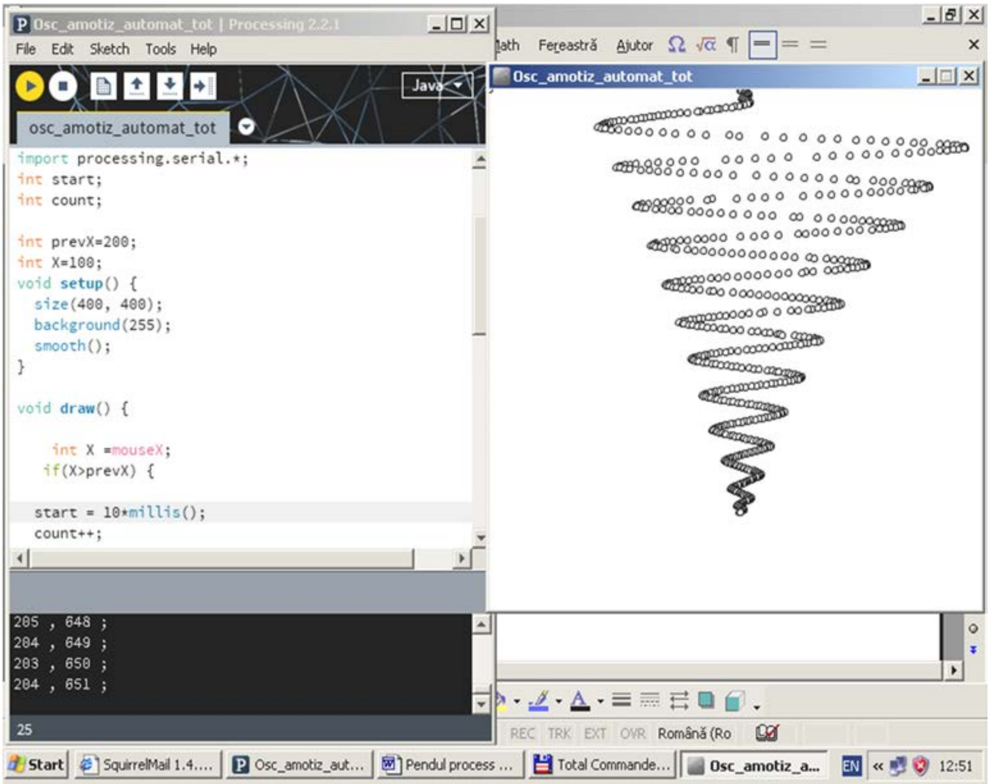

Fig. 2. Print screen of damped oscillations recorded with Processing. The data are automatically printed on the window workspace of Processing.

When the pendulum stops to oscillate, the acquisition of data is stopped. We can copy the data and analyze them with specific software like Origin or Kaleidagraph. Some theoretical considerations are necessarily in order to process the experimental data.

It is well known that the oscillations of the simple pendulum are determined by the tangential component of the gravity force acting on the body mass of the pendulum $G_t = -m \cdot g \cdot sin\phi$. For small angles of deviation $sin\phi \cong \phi$, and the displacements of the oscillating body from its equilibrium position is described by the equation $x = l \cdot sin\phi \cong l \cdot \phi$. Appling these approximations we can write $G_t \cong -m \cdot g \cdot \phi = -\frac{m \cdot g \cdot x}{l}$. ***l*** represents the length of the pendulum and ϕ the angle of deviation from the equilibrium position. In these conditions the force governing the oscillations is an elastic force of the form $\vec{F} = -k \cdot \vec{x}$, where the elastic constant *k* is given by the relation $k = \frac{m \cdot g}{l}$. We can apply all the equation of the elastic oscillator to describe the motion of the pendulum. If the oscillation of the pendulum is subject of friction with the coefficient μ , then the motion of the system is described by the equation:

$$m\frac{d^2x}{dt^2} = -kx - \mu\frac{dx}{dt} \tag{1}$$

The solution of this equation is a damped oscillation:

$$x = A \cdot \exp(-\alpha t) \cdot \sin(\omega t) \tag{2}$$

A represents the amplitude, α the attenuation coefficient and ω the pulsation of the damped oscillation. The relation existing between the parameters α, ω, *m* and *k* are described by the equations:

$$\alpha = \frac{\mu}{2m}, \qquad \omega^2 = \frac{k}{m} - \frac{\mu^2}{4m^2} = \frac{g}{l} - \frac{\mu^2}{4m^2} \tag{3}$$

The period of the damped oscilation $T = \frac{2\pi}{\omega}$ is greater than the period of the ideal oscillations

$$T_0 = \frac{2\pi}{\omega_0}, \qquad \text{where } \omega_0^2 = \frac{k}{m} = \frac{g}{l}, \qquad [6].$$

We are interested to measure and calculate the parameters of the real oscilations and compare them with those of the ideal pendulum. Experimentally we can measure the period and the amplitude of the damped oscillations, but we cannot directly measure the attenuation or the friction coefficients. These parameters can be obtained only by calculation. The amplitude of oscillations is affected by the attenuation coefficient α following an exponential law, [7]. Measuring the amplitudes $A(T_i)$ of damped oscillations at different instants T_i, we can calculate the attenuation coefficient α from the representation:

$$A(T_i) = A_o \, exp(-\alpha \, T_i) \qquad (4)$$

We can also calculate this coefficient from the logarithmic decrement of the oscillations, $\Lambda = \frac{A_2}{A_1}$. To do this we must measure the amplitudes A_1 and A_2 of two successive oscillations separated by a time interval equal with a period. Λ represents the rate of loss of mechanical energy W during a period, [8]. Taking into account the attenuation of the amplitude, we can find a direct relation between α and Λ.

$$\frac{W_2}{W_1} = exp(-2\alpha T) = \Lambda^2 \qquad \alpha = -\frac{ln\,\Lambda}{T} \qquad (5)$$

Knowing α, *m,* and *T* we can calculate the friction coefficient μ.

The experimental oscillations are presented in figure 3. From these data we measured the period *T* of the damped oscillations and we read the amplitudes $A(T_i)$ of the oscillations. We found the value $T = 1.23$ *s*. This value is smaller that the period of ideal oscillations T_0=*1.15 s* calculated with the relation $T_0 = 2\pi\sqrt{\frac{l}{g}}$. Using the equation (4) we calculated the attenuation coefficient α. We found the value α = *0.27* s^{-1}. The value of α was also calculated from the logarithmic decrement of the oscillations using equation (5). We took two successively values of amplitudes A_1 = *123 a.u* and A_2 = *94 a.u*. We obtained the value α = *0.268* s^{-1}. The values determined by both methods are in good agreement. Using this value of α, the measured values of *T* and *m*, we calculated the friction coefficient μ with the equation $\mu = 2m \cdot \alpha$. We found the value μ = *0.039*. We calculated also the elastic constant *k* with the equation (3). We found the value *k = 2.167 N/m*. To verify our result, we used the values α, μ, *m and T* to fit the experimental data with the equation (2). The result of the fit is shown in figure 3. We can see a good agreement between the experimental data and the simulation.

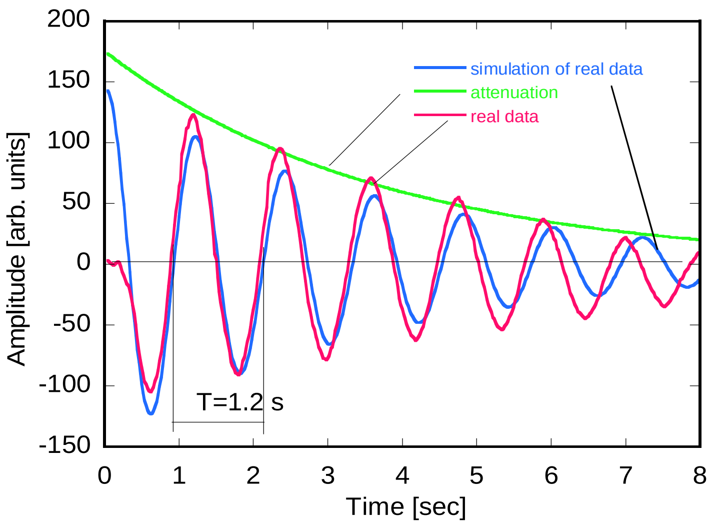

Fig. 3. Representation of real data, simulation of data with the calculated parameters and the theoretical attenuation

CONCLUSION

A simple method for digital data recording of oscillations of mechanical pendulum was presented. The method allows quantitative calculation of the parameters of the oscillations. The method is based on the use of very simple equipment, which can be ”*home made*”, and free software available on the internet. The method can be used for scientific experiments but also for didactic purpose, allowing the user to better understanding the principles of digital data acquisition.

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