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Dynamic behavior of a clamped circular plate and strain energy representation (part II)

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Abstract. *The paper presents the strain energy of the circular plate clamped all around through a relation obtained analytically and the graphic representation of the modal shapes and the maximum normalized strain energy along the x axis. Depending on the number of nodal circles s and the nodal diameters n, the maximum strain energy can be both in the center of the circular plate clamped all around, as well as the first ventral points from the center of the circular plate towards the outside of the plate.*

Keywords*: circular plate, Bessel functions, strain energy*

1. Introduction

In the second part of the paper, the authors present the strain energy of the circular plate clamped all around through a relation obtained analytically by using Bessel functions and the graphic representation of the modal shapes and the maximum normalized strain energy along the x axis, according to fig. 1 – 7 from first part of the paper.

In this context, the bibliographic references and the bibliography are identical to the one presented in the first part of the paper.

2. Strain energy

The strain energy of bending and twisting of the plate expressed in polar coordinates [11] is:

$$U = \frac{D}{2}\int_A \begin{pmatrix} \left(\frac{\partial^2 W}{\partial r^2} + \frac{1}{r}\frac{\partial W}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2}\right)^2 - \\ -2(1-\nu)\left\{\frac{\partial^2 W}{\partial r^2}\left(\frac{1}{r}\frac{\partial W}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2}\right) - \left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial W}{\partial \theta}\right)\right]^2\right\} \end{pmatrix} \mathrm{dA} \tag{1}$$

where,

dA=r·dr·dθ

D [Nm] is the flexural rigidity

ν is Poisson's ratio

r [m] is the plate radius

θ [rad] is the angle for polar coordinates

The strain energy is directly proportional to the second-order derivative of the modal function, respectively:

$$\begin{cases} \frac{\eth^2 W(r,\theta)_{n,s}}{\eth r^2} = -n^2\left[\frac{\eth^2 J_n\left(\lambda,\frac{r}{R}\right)}{\eth\lambda^2} - \frac{J_n(\lambda)}{I_n(\lambda)}\cdot\frac{\eth^2 I_n\left(\lambda,\frac{r}{R}\right)}{\eth\lambda^2}\right]cos(n\theta) \\ \frac{\eth^2 W(r,\theta)_{n,s}}{\eth r^2} = -n^2\left[\frac{\eth^2 J_n\left(\lambda,\frac{r}{R}\right)}{\eth\lambda^2} - \frac{J_n(\lambda)}{I_n(\lambda)}\cdot\frac{\eth^2 I_n\left(\lambda,\frac{r}{R}\right)}{\eth\lambda^2}\right]sin(n\theta) \end{cases} \quad (2)$$

The relations are used for the derivation of the Bessel functions:

$$\begin{cases} \frac{\eth J_n\left(\lambda,\frac{r}{R}\right)}{\eth\lambda} = -J_{n+1}(\lambda) + \frac{n}{\lambda}J_n(\lambda) = J_{n-1}(\lambda) - \frac{n}{\lambda}J_n(\lambda) \\ \frac{\eth I_n\left(\lambda,\frac{r}{R}\right)}{\eth\lambda} = I_{n+1}(\lambda) + \frac{n}{\lambda}I_n(\lambda) = I_{n-1}(\lambda) - \frac{n}{\lambda}I_n(\lambda) \end{cases} \quad (3)$$

and the strain energy function becomes:

$$\begin{cases} \frac{\eth^2 W(r,\theta)_{n,s}}{\eth r^2} = \begin{bmatrix} \frac{n^2-n-\lambda^2}{\lambda^2}J_n\left(\lambda,\frac{r}{R}\right) + \frac{1}{\lambda}J_{n+1}\left(\lambda,\frac{r}{R}\right) - \\ -\frac{J_n(\lambda)}{I_n(\lambda)}\left(\frac{n^2-n+\lambda^2}{\lambda^2}I_n\left(\lambda,\frac{r}{R}\right) - \frac{1}{\lambda}I_{n+1}\left(\lambda,\frac{r}{R}\right)\right) \end{bmatrix} cos(n\theta) \\ \frac{\eth^2 W(r,\theta)_{n,s}}{\eth r^2} = \begin{bmatrix} \frac{n^2-n-\lambda^2}{\lambda^2}J_n\left(\lambda,\frac{r}{R}\right) + \frac{1}{\lambda}J_{n+1}\left(\lambda,\frac{r}{R}\right) - \\ -\frac{J_n(\lambda)}{I_n(\lambda)}\left(\frac{n^2-n+\lambda^2}{\lambda^2}I_n\left(\lambda,\frac{r}{R}\right) - \frac{1}{\lambda}I_{n+1}\left(\lambda,\frac{r}{R}\right)\right) \end{bmatrix} sin(n\theta) \end{cases} \quad (4)$$

It can be seen from (4) that the strain energy function is a surface function of r/R and the angle θ.

3. Results

The maximum strain energy from the first relation (4) is obtained along the x direction with de notation from fig. 1 – 7 from first part of the paper.

For the dimensionless wave numbers presented in tab. 1 (part I of the paper), below, in figures 1 - 7 are presented the modal shapes and the maximum strain energy along the x direction.

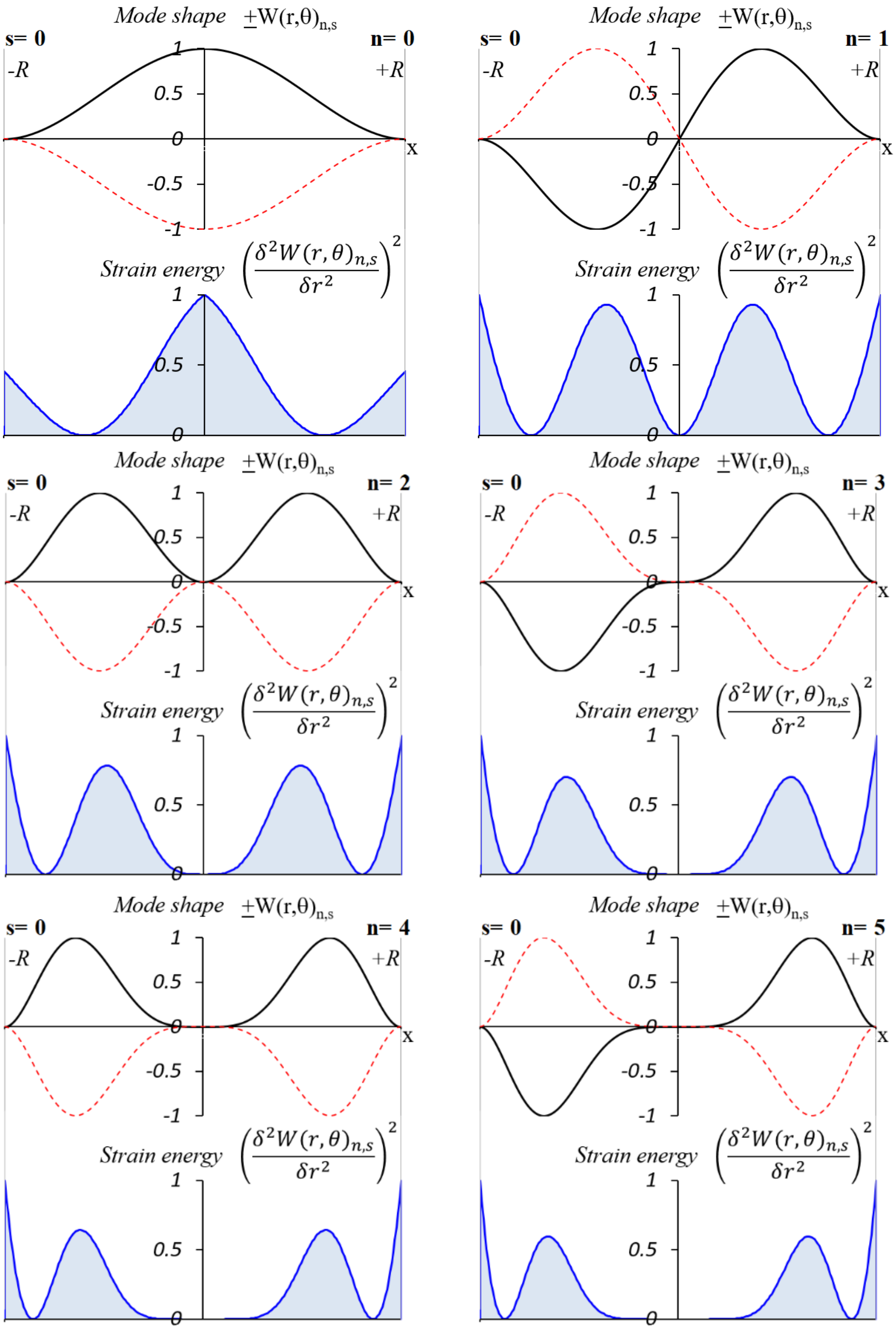

Figure 1. Normalized mode shape and strain energy for s=0 and n=0, …, 5.

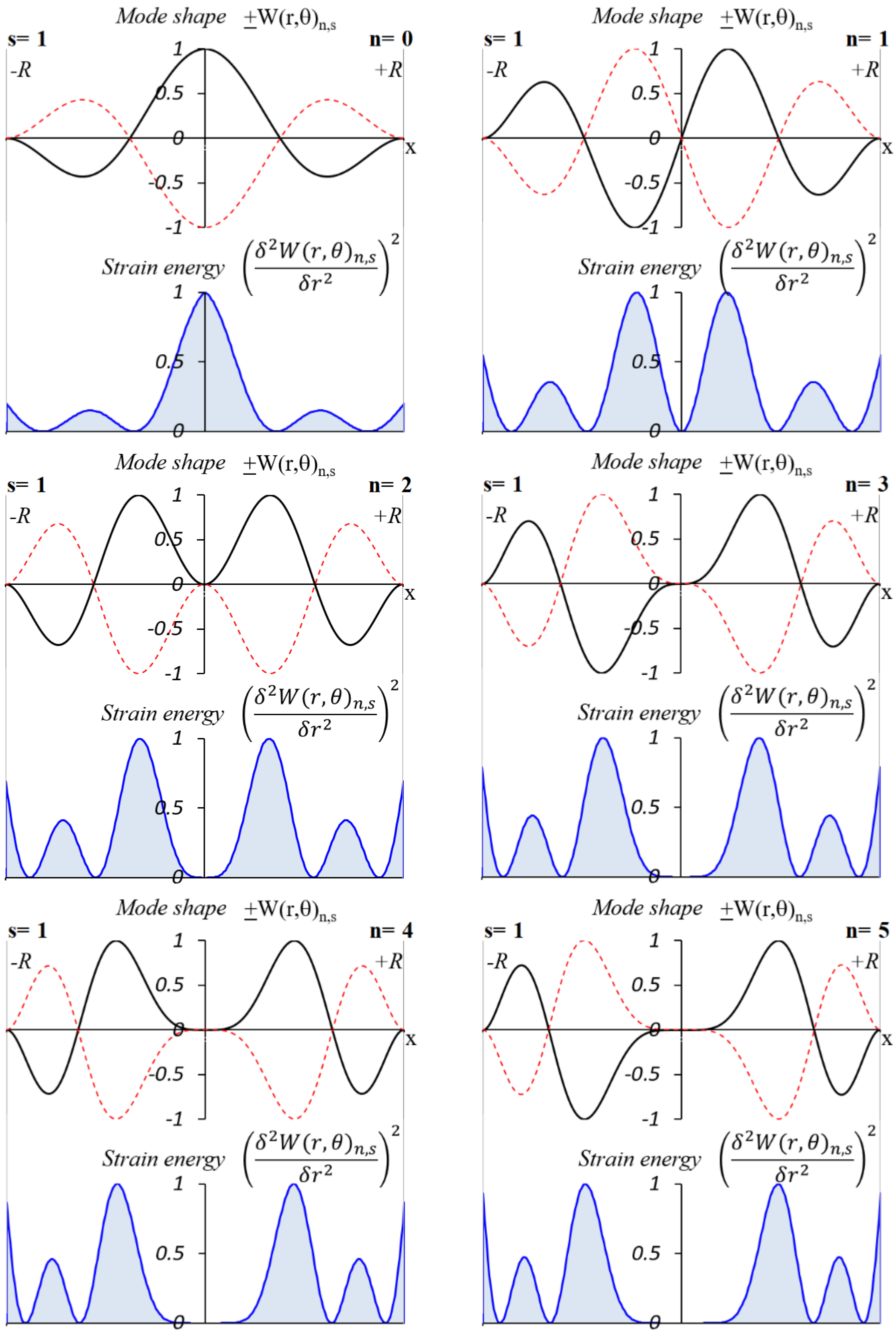

Figure 2. Normalized mode shape and strain energy for s=1 and n=0, …, 5.

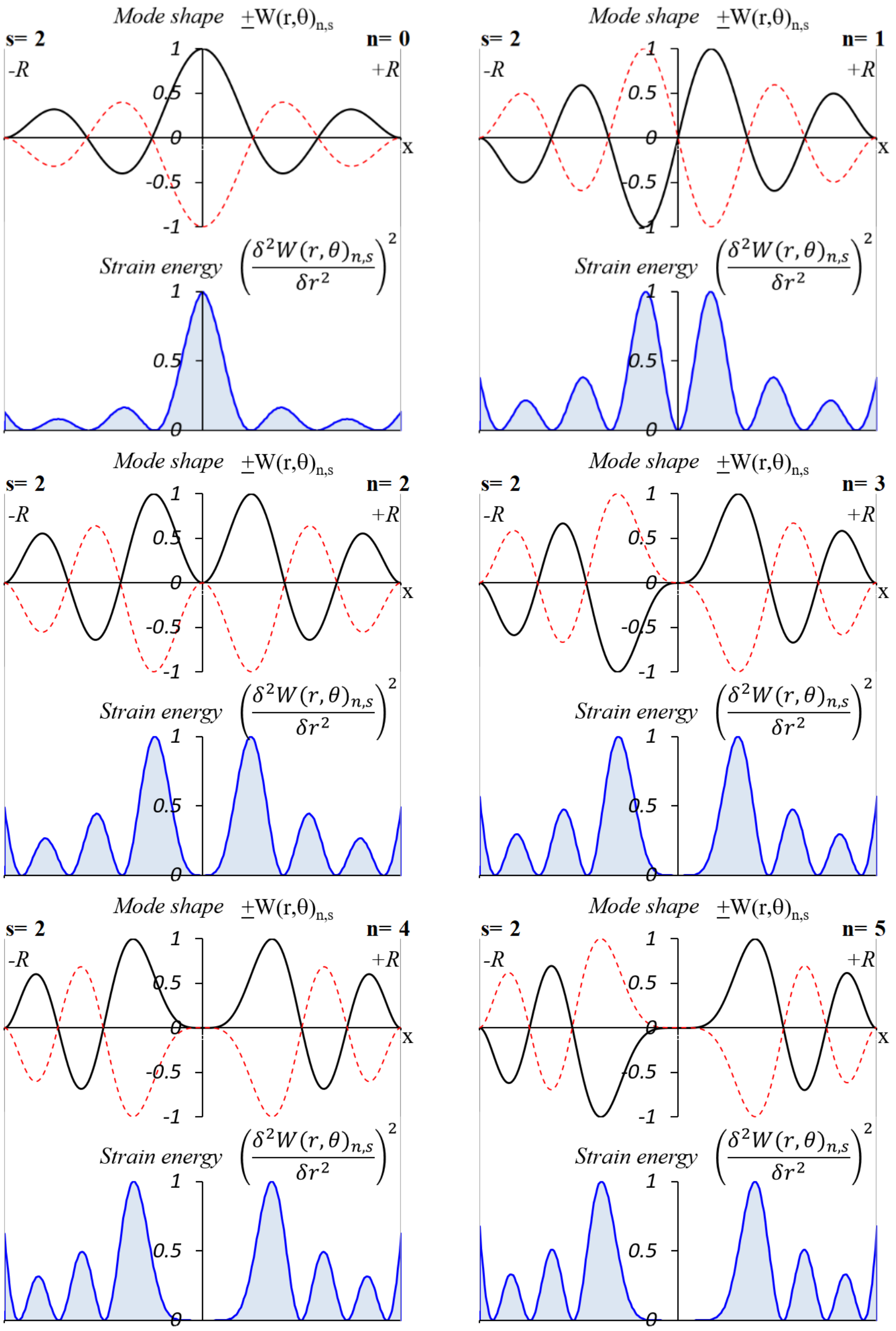

Figure 3. Normalized mode shape and strain energy for s=2 and n=0, …, 5.

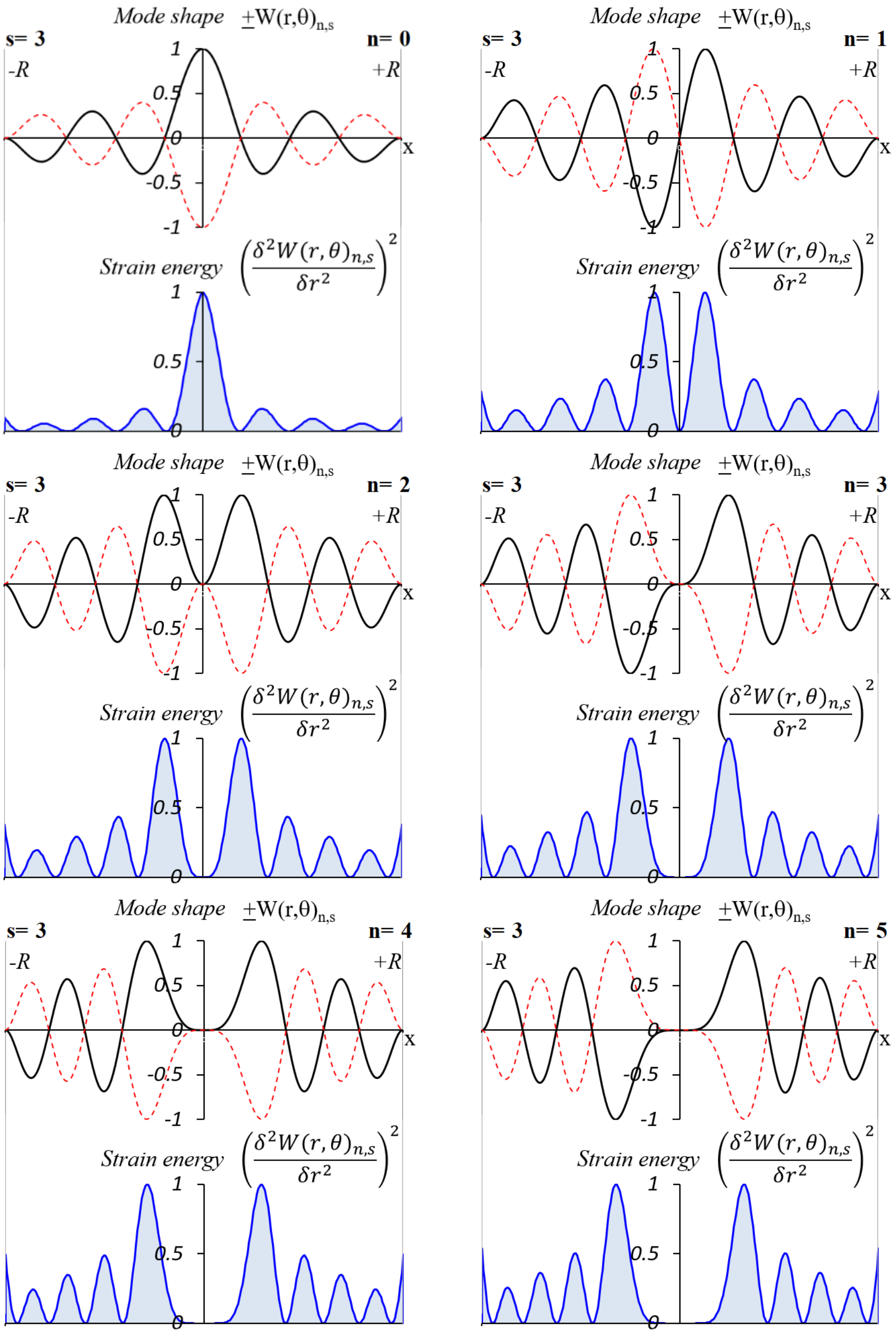

Figure 4. Normalized mode shape and strain energy for s=3 and n=0, …, 5.

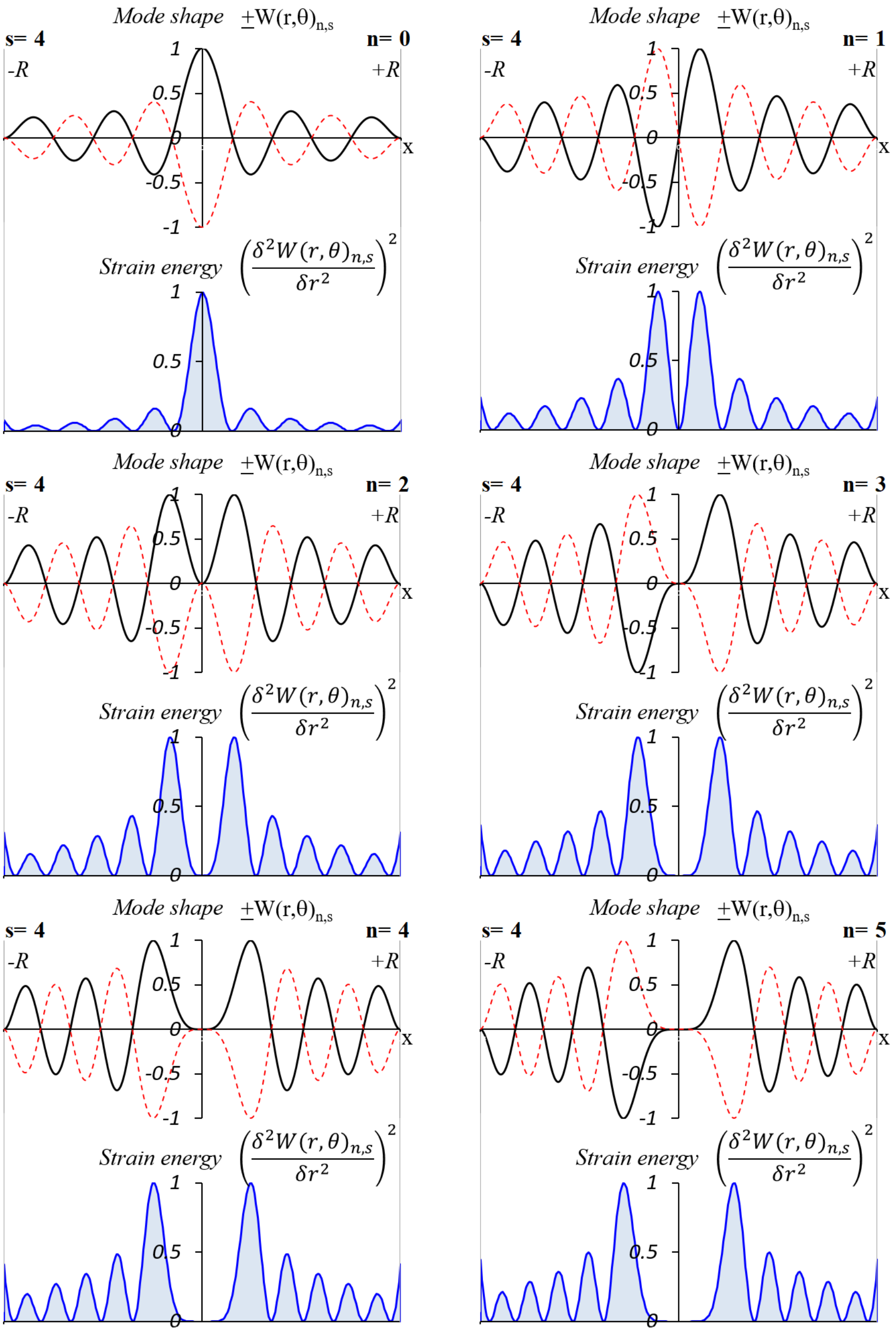

Figure 5. Normalized mode shape and strain energy for s=4 and n=0, …, 5.

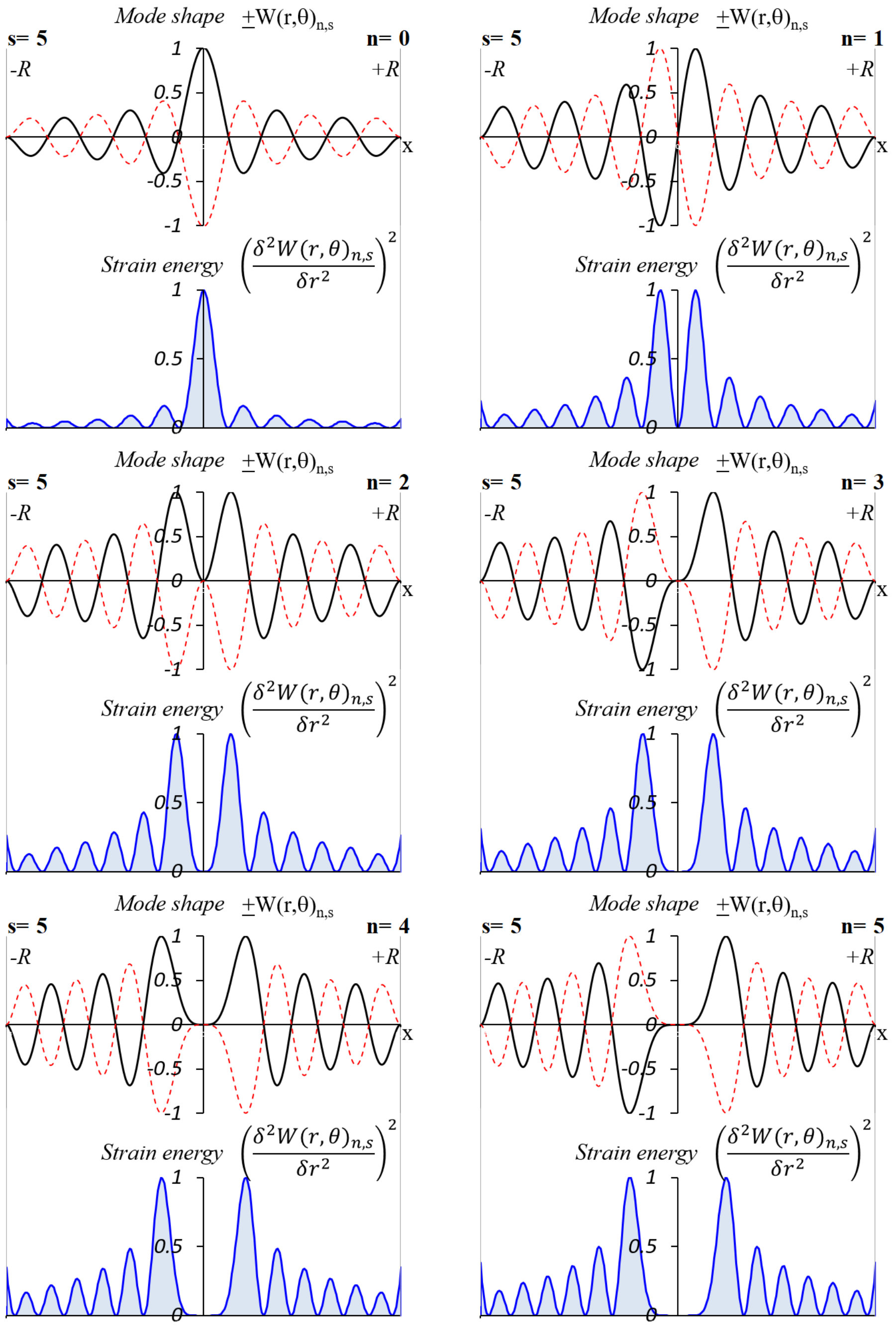

Figure 6. Normalized mode shape and strain energy for s=5 and n=0, …, 5.

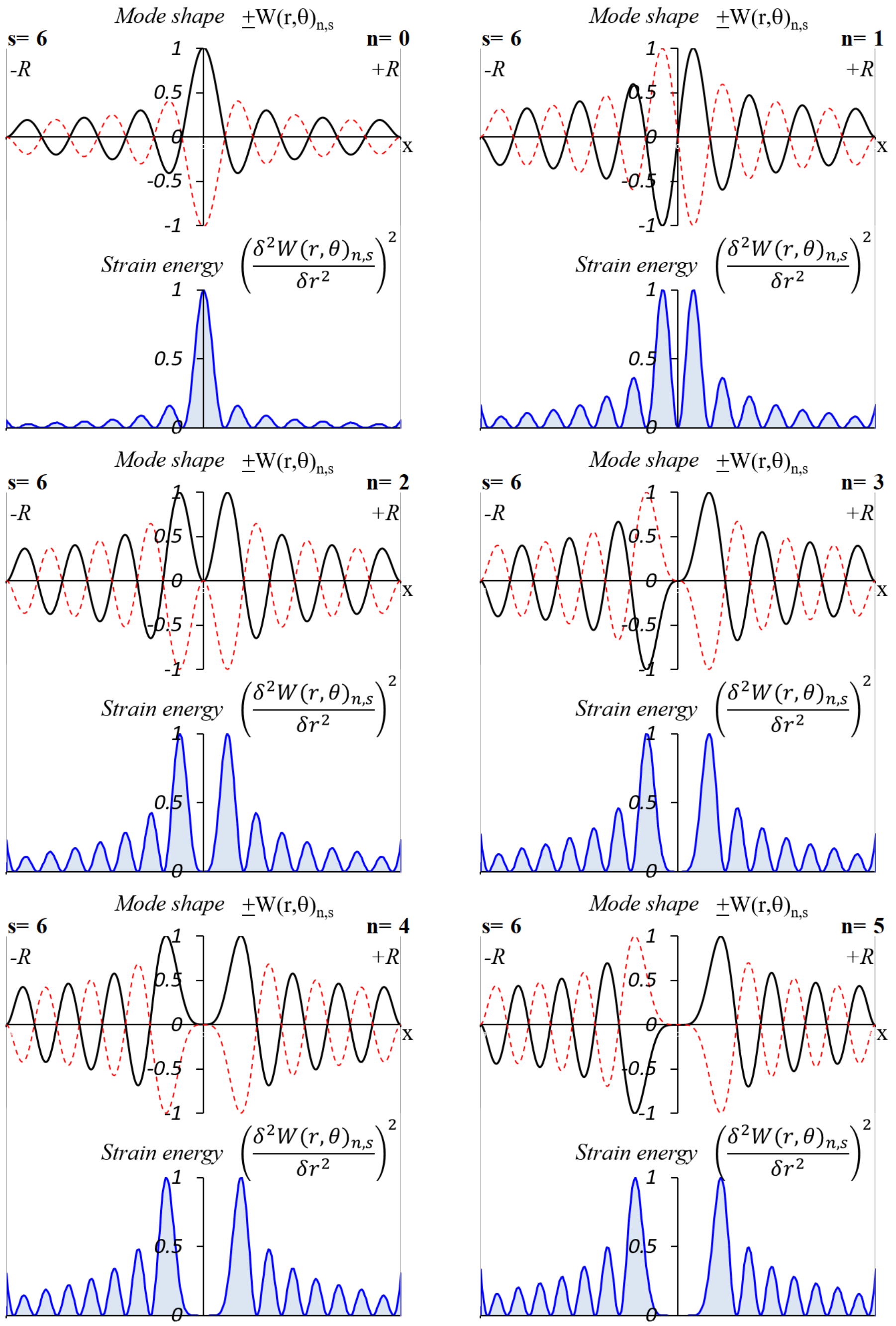

Figure 7. Normalized mode shape and strain energy for s=6 and n=0, …, 5.

Fig. 1 present the first six vibration modes and normalized strain energy for nodal circle s=0 and nodal diameters n=0, 1, 2, 3, 4 and 5 on x direction.

Fig. 2 present the first six vibration modes and normalized strain energy for nodal circle s=1 and nodal diameters n=0, 1, 2, 3, 4 and 5 on x direction.

Fig. 3 present the first six vibration modes and normalized strain energy for nodal circle s=2 and nodal diameters n=0, 1, 2, 3, 4 and 5 on x direction.

Fig. 4 present the first six vibration modes and normalized strain energy for nodal circle s=3 and nodal diameters n=0, 1, 2, 3, 4 and 5 on x direction.

Fig. 5 present the first six vibration modes and normalized strain energy for nodal circle s=4 and nodal diameters n=0, 1, 2, 3, 4 and 5 on x direction.

Fig. 6 present the first six vibration modes and normalized strain energy for nodal circle s=5 and nodal diameters n=0, 1, 2, 3, 4 and 5 on x direction.

Fig. 7 present the first six vibration modes and normalized strain energy for nodal circle s=6 and nodal diameters n=0, 1, 2, 3, 4 and 5 on x direction.

4. Conclusion

The paper presents the normalized vibration modes and strain energy for a circular plate clamped all around obtained for the maximum value of strain energy on x direction taking into consideration the first relationship (4). Using the Bessel functions of the first kind, the strain energy function was analytically determined.

The normalized mode shape and strain energy is illustrated in fig. 1 – 7 for the following dimensionless wave numbers $\lambda_{n,s}^2$: nodal circles s=0, 1, ..., 6 and nodal diameters n=0, 1, .., 5.

From the analysis of figures 1 - 7 it can be found that the maximum normalized strain energy is in the center of the circular plate clamped all around, for the nodal diameter n=0 regardless of the number of nodal circles s. For values of nodal diameters n>0 the strain energy becomes zero in the center of the plate.

For s=0 and n>0, the strain energy is maximum at the clamped area of the circular plate.

For s>0 and n>0, the deformation energy is maximum at the first ventral point from the center of the plate, compared to the case of the doubly supported beam where the maximum strain energy is right at the clamped end. As the number of nodal diameters n increases, the maximum strain energy keeps moving away from the center of the circular plate.

Taking into account the 3D representation of the mode shapes (presented in part I), it is found that for n=1 we have one diameter of inflection, for n=2 we have 4 diameters of inflection, for n=3 there are 6 diameters of inflection, and so on, so we can say that for n >1, the number of inflection diameters is equal to 2n. The number of inflection circles is equal to the number of nodal circles s.

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