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Heat absorption and Hall current effects on unsteady MHD flow past an inclined plate

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In the present paper, we study the effect of heat absorption on unsteady flow of a viscous, incompressible, electrically conducting fluid past an impulsively started inclined plate with variable wall temperature and mass diffusion in the presence of transversely applied uniform magnetic field and Hall current. Earlier we analyzed the effects of radiation and chemical reaction on MHD flow past a vertical plate with variable temperature and mass diffusion. We had obtained the results which were in agreement with the desired flow phenomenon. To study further, we are changing the model by considering heat absorption on fluid, and changing the geometry of the model. Here in this paper we are considering the plate positioned inclined from vertically plane and impulsively started with velocity u_0. The temperature of plate and the concentration level near the plate is increase linearly with time. The governing equations involved in the present analysis are solved by the Laplace-transform technique. The results obtained have been analyzed with the help of graphs drawn for different parameters like thermal Grashof number, mass Grashof Number, Prandtl number, permeability parameter, Hall current parameter, heat absorption parameter, magnetic field parameter and Schmidt number. The numerical values obtained for skin-friction and Nusselt number have been tabulated. The results are found to be in a good agreement and the data obtained is in concurrence with the actual MHD fluid flow phenomenon.

Keywords: *MHD flow, heat absorption, inclined plate, variable temperature, mass diffusion, Hall current.*

1. Introduction

The effect of heat absorption on MHD with heat and mass transfer is of great importance in many applications such as development of metal waste from spent nuclear fuel, fluids undergoing exothermic and endothermic chemical reaction, convection in Earth's mantle, post-accident heat removal, fire and combustion modeling etc. Hall effect on MHD mixed convective flow of a viscous incompressible fluid past a vertical porous plate immersed in porous medium with heat source was analyzed by Sharma et.al [11]. MHD-conjugate heat transfer analysis for a vertical flat plate in presence of viscous dissipation and heat generation was studied by Mamun et. al [5]. Shit and Haldar [8] have worked on combined effects of thermal radiation and Hall current on MHD free-convective flow and mass transfer over a stretching sheet with variable viscosity. Radiation effect on MHD free convection flow along vertical flat plate in presence of Joule heating and heat generation was proposed by Alia et.al [1]. Seth et.al [10] have investigated numerical solution of unsteady hydromagnetic natural convection flow of heat absorbing fluid past an impulsively moving vertical plate with ramped temperature. Tanvir and Alam [12] have worked on finite difference solution of MHD mixed convection flow with heat generation and chemical reaction. Unsteady hydromagnetic flow of a heat absorbing dusty fluid past a permeable vertical plate with ramped temperature was analyzed by Das et.al [2]. Stanford and Gerald [9] have studied on a new numerical analysis of the Hall effect on MHD flow and heat transfer over an unsteady stretching permeable surface in the presence of thermal radiation and heat source. Chemical reaction effect on unsteady MHD flow past an impulsively started oscillating inclined plate with variable temperature and mass diffusion in the presence of Hall current was presented by Rajput anf Kumar [6]. Further, Rajput and Kumar [7] have worked on effects of radiation and chemical reaction on MHD flow past a vertical plate with variable temperature and mass diffusion. Kumar along with Bansal [3] have considered unsteady flow past on vertical cylinder in the presence of an inclined magnetic field and chemical reaction. Chemical reaction effect on MHD flow past an impulsively started vertical cylinder with variable temperature and mass diffusion was studied by Kumar et.al [4]. This paper deals with an analysis of effects of heat absorption and Hall current on unsteady flow past an impulsively started inclined plate in the presence of transversely applied uniform magnetic field with heat and mass transfer. The problem is solved analytically using the Laplace Transform technique. A selected set of graphical results illustrating the effects of various parameters involved in the problem are presented and discussed. The numerical values of skin-friction have been tabulated.

2. Mathematics Analysis.

The geometric flow model of the problem is shown in Figure-1. The unsteady flow of an electrically conducting, viscous, incompressible and heat absorbing fluid past an impulsively started inclined plate with variable wall temperature and mass diffusion in the presence of Hall current has been considered. The x axis is taken along the vertical plane and z axis is normal to it. Thus the z axis lies in the horizontal plane. The plate is inclined at angle α from vertical. The magnetic field B_0 of uniform strength is applied perpendicular to the flow. Further, the angle of inclination changes then the magnetic field changes its direction, such that it always remains perpendicular to it. Thus the direction of magnetic field is tied with the plate. Initially it has been considered that the plate as well as the fluid is at the same temperature T_∞. The species concentration in the fluid is taken as C_∞. At time t > 0 the plate starts moving with velocity u_0 in its own plane and temperature of the plate are raised to T_w. The concentration C near the plate is raised linearly with respect to time.

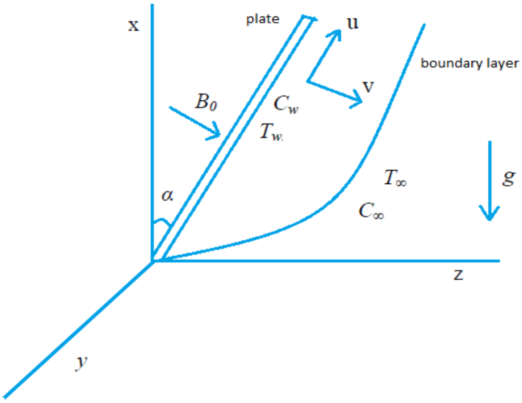

Figure 1.

Then under these assumptions and the Boussinesq's approximation, the flow is governed by the following system of equations:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial z^2} + g\beta Cos\alpha\,(T - T_\infty) + g\beta^* Cos\alpha\,(C - C_\infty) - \frac{\sigma B_0^2 (u + mv)}{\rho(1+m^2)}, \tag{1}$$

$$\frac{\partial v}{\partial t} = \upsilon \frac{\partial^2 v}{\partial z^2} + \frac{\sigma B_0^2 (mu - v)}{\rho(1+m^2)}, \tag{2}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial z^2}, \tag{3}$$

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial z^2} - Q(T - T_\infty). \tag{4}$$

The following initial and boundary conditions are

$$\left.\begin{array}{l} t \le 0: u=0,\ v=0,\ T=T_\infty,\ C=C_\infty,\ \text{ for all } z, \\ t>0: u=u_0,\ v=0,\ T=T+(T_w-T_\infty)\dfrac{u_0^2 t}{\upsilon},\ C=C_\infty+(C_w-C_\infty)\dfrac{u_0^2 t}{\upsilon},\ \text{at} z=0, \\ u \to 0,\ v=0,\ T\to T_\infty,\ C\to C_\infty \ \text{ as } z\to\infty. \end{array}\right\} \quad (5)$$

Here u and v are the primary and secondary velocities, g- the acceleration due to gravity, β- volumetric coefficient of thermal expansion, t- time, $m(=\omega_e\tau_e)$ is the Hall current parameter with ω_e - cyclotron frequency of electrons and τ_e- electron collision time, T- temperature of the fluid, β^*- volumetric coefficient of concentration expansion, C- species concentration in the fluid, ν- the kinematic viscosity, ρ- the density, C_p- the specific heat at constant pressure, k- thermal conductivity of the fluid, D- the mass diffusion coefficient, T_w- temperature of the plate at $z=0$, Q- heat absorption coefficient C_w - species concentration at the plate z= 0, B_0- the uniform magnetic field, σ - electrical conductivity.

The following non-dimensional quantities are introduced to transform equations (1), (2), (3) and (4) into dimensionless form:

$$\left.\begin{array}{l} \bar{z}=\dfrac{zu_0}{\upsilon},\ \bar{u}=\dfrac{u}{u_0},\ \bar{v}=\dfrac{v}{u_0},\ \theta=\dfrac{(T-T_\infty)}{(T_w-T_\infty)},\ S_c=\dfrac{\upsilon}{D},\ \mu=\rho\upsilon, \\ P_r=\dfrac{\mu C_p}{k}, H=\dfrac{Q\upsilon}{u_0^2\rho C_p}, G_r=\dfrac{g\beta\upsilon(T_w-T_\infty)}{u_0^3},\ M=\dfrac{\sigma B_0^2\upsilon}{\rho u_0^2}, \\ G_m=\dfrac{g\beta^*\upsilon(C_w-C_\infty)}{u_0^3},\ \bar{C}=\dfrac{(C-C_\infty)}{(C_w-C_\infty)},\ \bar{t}=\dfrac{tu_0^2}{\upsilon}. \end{array}\right\} \quad (6)$$

The symbols in dimensionless form are as under:
$\bar{u}$ is the primary velocity, $\bar{v}$ - the secondary velocity, $\bar{t}$ - time, θ- the temperature, $\bar{C}$ - the concentration, G_r - thermal Grashof number, G_r- mass Grashof number, μ- the coefficient of viscosity, P_r- the Prandtl number, S_c- the Schmidt number, H- heat absorption coefficient, M- the magnetic parameter.

Thus the model becomes:

$$\frac{\partial\bar{u}}{\partial\bar{t}}=\frac{\partial^2\bar{u}}{\partial\bar{z}^2}+G_r Cos\alpha\,\theta+G_m Cos\alpha\,\bar{C}-\frac{M(\bar{u}+m\bar{v})}{(1+m^2)}, \quad (7)$$

$$\frac{\partial\bar{v}}{\partial\bar{t}}=\frac{\partial^2\bar{u}}{\partial\bar{z}^2}+\frac{M(m\bar{u}-\bar{v})}{(1+m^2)}, \quad (8)$$

$$\frac{\partial \overline{C}}{\partial \bar{t}} = \frac{1}{S_c} \frac{\partial^2 \overline{C}}{\partial \bar{z}^2}, \tag{9}$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial \bar{z}^2} - H\theta. \tag{10}$$

The corresponding boundary conditions become:

$$\left.\begin{array}{ll} \bar{t} \le 0 : \bar{u} = 0,\ \bar{v} = 0,\ \theta = 0,\ \overline{C} = 0, & \text{for all } \bar{z}, \\ \bar{t} > 0 : \bar{u} = 1,\ \bar{v} = 0,\ \theta = \bar{t},\ \overline{C} = \bar{t}, & \text{at } \bar{z} = 0, \\ \bar{u} \to 0,\ \bar{v} \to 0,\ \theta \to 0,\ \overline{C} \to 0, & \text{as } \bar{z} \to \infty. \end{array}\right\} \tag{11}$$

Dropping bars in the above equations, we get:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial z^2} + G_r Cos\alpha\, \theta + G_m Cos\alpha\, C - \frac{M(u + mv)}{(1 + m^2)}, \tag{12}$$

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial z^2} + \frac{M(mu - v)}{(1 + m^2)}, \tag{13}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2}, \tag{14}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial z^2} - H\theta. \tag{15}$$

The boundary conditions are:

$$\left.\begin{array}{ll} t \le 0 : u = 0, v = 0, \theta = 0, C = 0, & \text{for all } z, \\ t > 0 : u = 1, v = 0,\ \theta = t,\ C = t, & \text{at } z = 0, \\ u \to 0,\ v \to 0,\ \theta \to 0,\ C \to 0, & \text{as } z \to \infty. \end{array}\right\} \tag{16}$$

Combining equations (12) and (13), the flow model becomes:

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2} + G_r Cos\alpha\, \theta + G_m Cos\alpha\, C - q\left(\frac{M(1 - im)}{1 + m^2}\right), \tag{17}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2}, \tag{18}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial z^2}-H\theta. \tag{19}$$

Finally, the boundary conditions become:

$$\left.\begin{array}{lll} t\le 0: q=0,\theta=0,C=0, & \text{for all } z, \\ t>0: q=1,\theta=t,C=t, & \text{at } z=0, \\ q\to 0,\ \theta\to 0,\ C\to 0, & \text{as } z\to\infty. \end{array}\right\} \tag{20}$$

Here q= u + i v,

The dimensionless governing equations (17) to (19), subject to the boundary conditions (20), are solved by the usual Laplace - transform technique. The solutions obtained are as under:

$$C=t\left\{(1+\frac{z^2S_c}{2t})erfc[\frac{\sqrt{S_c}}{2\sqrt{t}}]-\frac{z\sqrt{S_c}}{\sqrt{\pi\sqrt{t}}}e^{-\frac{z^2}{4t}}S_c\right\},$$

$$\theta=-\frac{1}{4Hi}e^{-zi\sqrt{-HP_r}}\left\{-2i\sqrt{-H}t(A_1-e^{-zi\sqrt{-HP_r}}A_2-2)+z\sqrt{P_r}(A_1+e^{-zi\sqrt{-HP_r}}A_2-2)\right\}$$

$$\begin{aligned}
q=&\frac{1}{2}e^{-\sqrt{M(1-im)(1+m^2)^{-1}}z}A_{33}+\frac{G_rCos\alpha}{4}\left(\left(M(1-im)(1+m^2)^{-1}\right)-HP_r\right)^{-2}[(M(1\\
&-im)(1+m^2)^{-1}t+P_r-HP_rt-1)(e^{-\sqrt{M(1-im)(1+m^2)^{-1}}z}A_3-e^{-\sqrt{-HP_r}zi}A_{11})\\
&+e^{-\sqrt{M(1-im)(1+m^2)^{-1}}z}A_4(\sqrt{M(1-im)(1+m^2)^{-1}}-HP_r\left(M(1-im)(1+m^2)^{-1}\right)^{-1/2})\\
&+A_{14}(1-P_r)(A_5-A_{12})-\frac{1}{2i\sqrt{-H}}ze^{-\sqrt{-HP_r}zi}A_{10}\sqrt{P_r}(-HP_r+\left(M(1-im)(1+m^2)^{-1}\right))]\\
&+\frac{G_mCos\alpha}{4}\left(M(1-im)(1+m^2)^{-1}\right)^{-2}[2A_6e^{-\sqrt{a}z}(1-M(1-im)(1+m^2)^{-1}t)\\
&+e^{-\sqrt{\left(M(1-im)(1+m^2)^{-1}\right)}z}(z\sqrt{M(1-im)(1+m^2)^{-1}}A_8+2A_9S_c)+2A_{15}A_7(1-S_c)]\\
&+\frac{G_mCos\alpha}{2\sqrt{\pi}}\left(M(1-im)(1+m^2)^{-1}\right)^{-2}[2\left(M(1-im)(1+m^2)^{-1}\right)z\sqrt{tS_c}e^{-\frac{z^2S_c}{4t}}\\
&+A_{16}\sqrt{\pi}(M(1-im)(1+m^2)^{-1}z^2S_c+2tM(1-im)(1+m^2)^{-1}+2S_c-2)\\
&+A_{13}A_{15}\sqrt{\pi}(1-S_c)]
\end{aligned}$$

The expressions for the constants involved in the above equations are given in the appendix.

Skin friction

The dimensionless skin friction at the plate z=0:

$$\left(\frac{dq}{dz}\right)_{z=0} = \tau_x + i\tau_y \ ,$$

Separating real and imaginary part in $\left(\frac{dq}{dz}\right)_{z=0}$, the dimensionless skin friction component $\tau_x = \left(\frac{du}{dz}\right)_{z=0}$ and $\tau_y = \left(\frac{dv}{dz}\right)_{z=0}$ can be computed.

Nusselt number

The dimensionless Nusselt number at the plate *z=0* is given by:

$$Nu = \left(\frac{\partial\theta}{\partial z}\right)_{z=0} = -t\sqrt{HP_r} + \frac{1}{4\sqrt{H}}((-2 + 2erfc[\sqrt{Ht}])\sqrt{P_r} - 2\sqrt{H}t(\frac{2\exp(Ht)\sqrt{P_r}}{\sqrt{\pi t}}$$
$$-2erfc[\sqrt{Ht}]\sqrt{HP_r})$$

3. Results and Discussion

The velocity profiles for different parameters like thermal Grashof number (*Gr*), magnetic field parameter (*M*), Hall parameter (*m*), Prandtl number (*Pr*) and Heat absorption parameter (*H*) are shown in figures 1.1 to 2.8. The fluid flow with high velocity when plate is vertical and low velocity when plate is horizontal, It is observed from figures 1.1 and 2.1 that the primary and secondary velocities of fluid decrease when the angle of inclination (α) is increased. This is due to facts that fluid velocity depends on gravity components (*gCosα*). Figures 1.2, 2.2, 1.3 and 2.3 show the buoyancy effect, and it is observed that both the primary and secondary velocities increase on increasing thermal Grashof number G_r and mass Grashof number G_m. Therefore, it concludes that buoyancy force tends to accelerate primary and secondary velocities. Also, if Hall current parameter *m* is increased then *u* increases, while *v* gets decreased (figures 1.4 and 2.4). The influence of magnetic field in a electrically conducting fluid has established a force which is known as Lorentz force and that force perform against the main flow. So these kind of resisting forces slow down the primary velocity of fluid as detected from figures 1.5 and 2.5 that the effect of increasing values of the parameter *M* results in decreasing *u* and increasing *v*. It is deduced that when heat absorption parameter *H* is increased then the velocity *u* is increased, while velocity *v* get decreased (figures 1.6 and 2.6). Further, it is observed that velocities decrease when Prandtl number and Schmidt number are increased (figures 1.7, 2.7, 1.8 and 2.8). In actual sense, the increase of Sc means decrease of molecular diffusivity *(D)*. This means the process of diffusion will decrease.

Skin friction is given in table1. The value of τ_x increases with the increase in Hall current parameter, Prandtl number, thermal Grashof number, mass Grashof Number, heat absorption parameter and time, and it decreases with the angle of inclination of plate, the magnetic field and Schmidt number. Similar effect is observed with τ_y, except the angle of inclination of plate, magnetic field, and time, in which case τ_y increases with the angle of inclination of plate, magnetic field parameter, and decreases with time. Nusselt number is given in table2. The value of Nu decreases with increase in Prandtl number, heat absorption parameter and time.

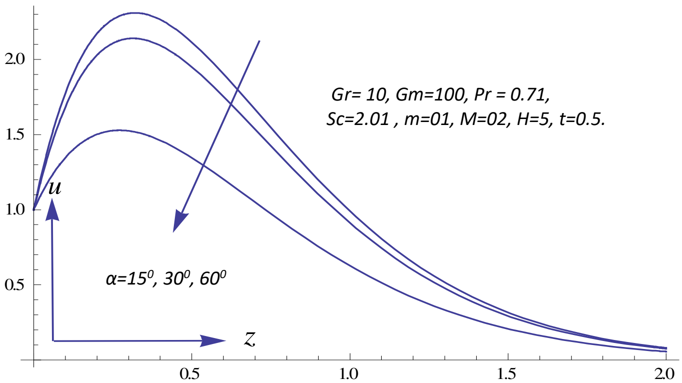

Figure 1.1. Velocity u for different values of α

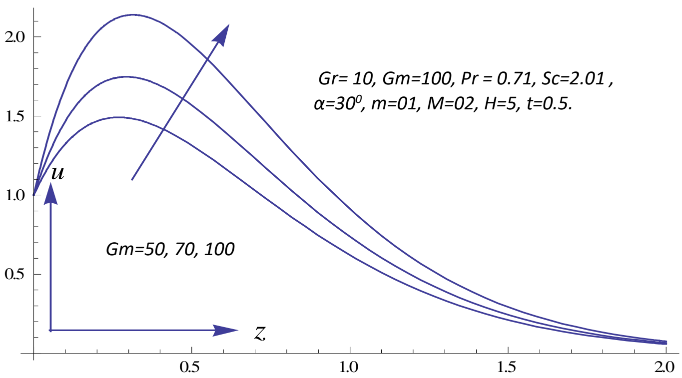

Figure 1.2. Velocity u for different values of *Gm*.

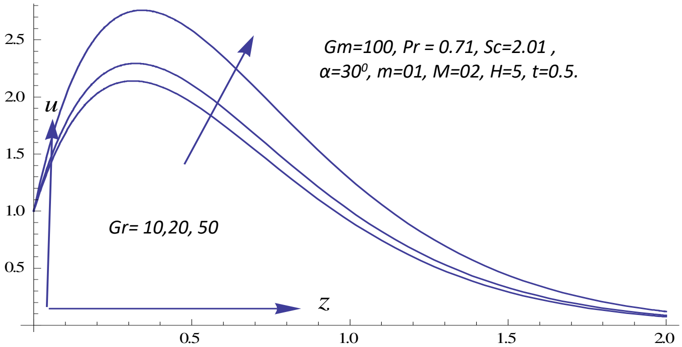

Figure 1.3. Velocity u for different values of *Gr*.

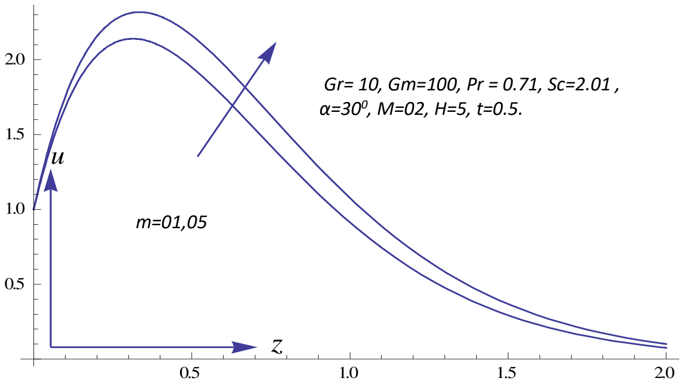

Figure 1.4. Velocity u for different values of *m*.

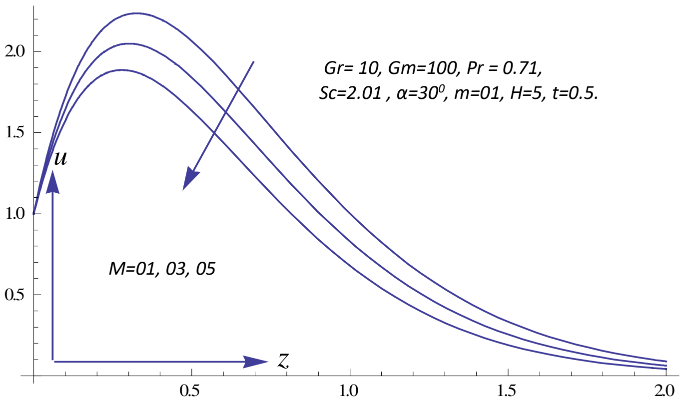

Figure 1.5. Velocity u for different values of *M*.

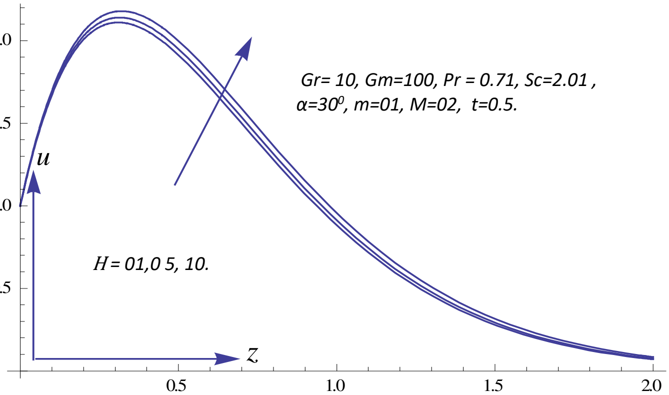

Figure 1.6. Velocity u for different values of *H*.

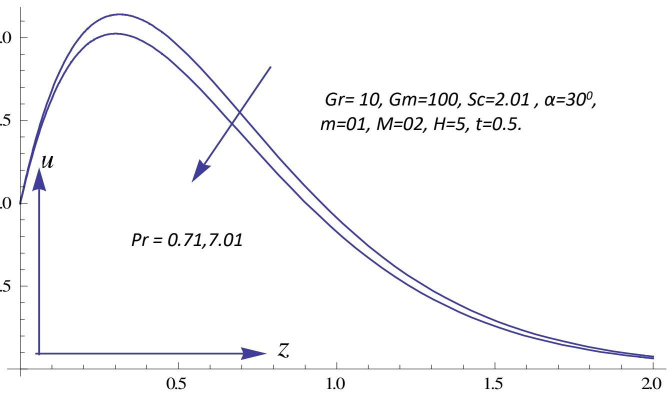

Figure 1.7. Velocity u for different values of *Pr*.

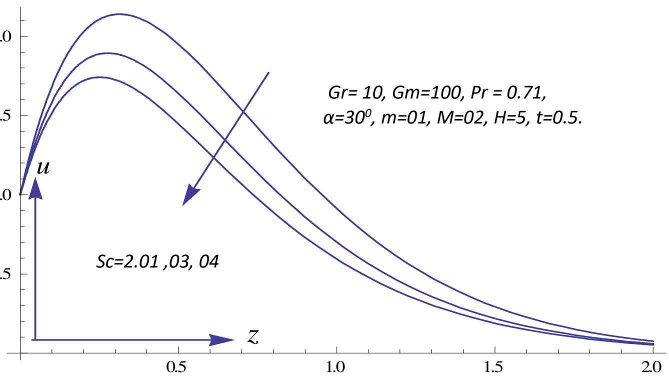

Figure 1.8. Velocity u for different values of *Sc*.

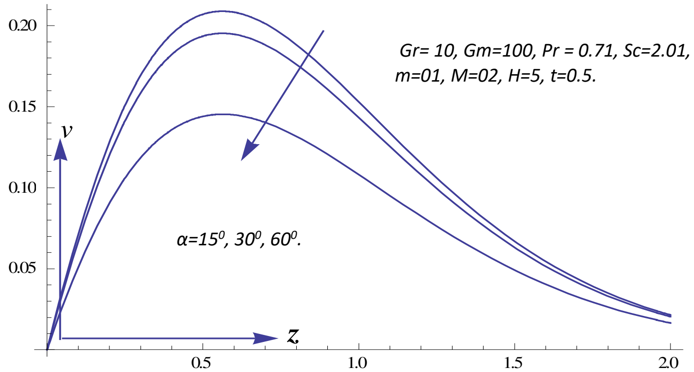

Figure 2.1. Velocity v for different values of α

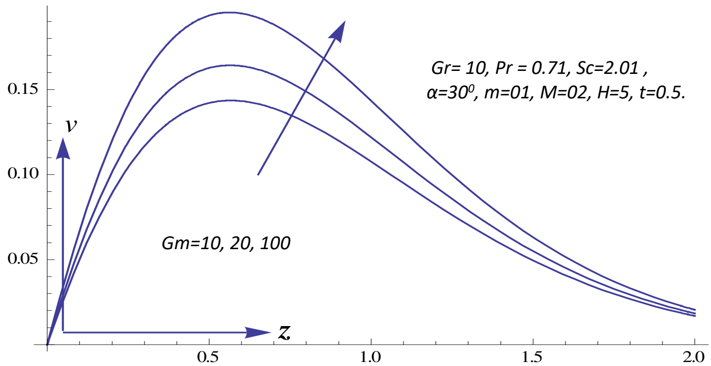

Figure 2.2. Velocity v for different values of *Gm.*

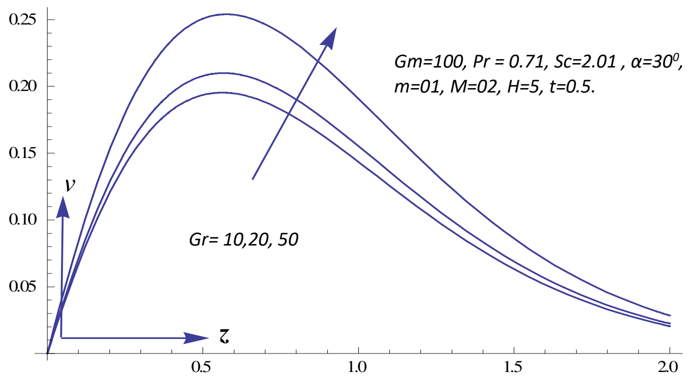

Figure 2.3. Velocity v for different values of *Gr*.

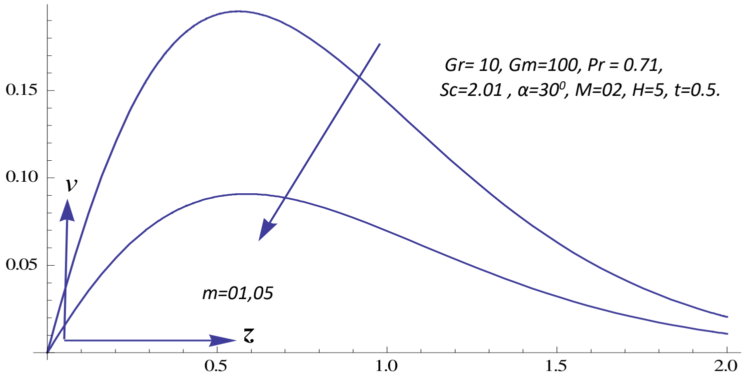

Figure 2.4. Velocity v for different values of *m*.

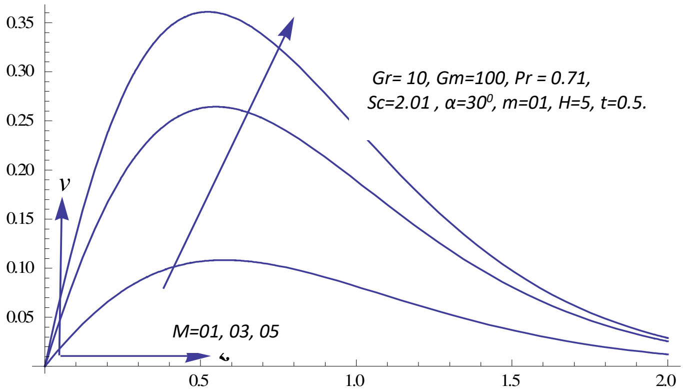

Figure 2.5. Velocity v for different values of *M*.

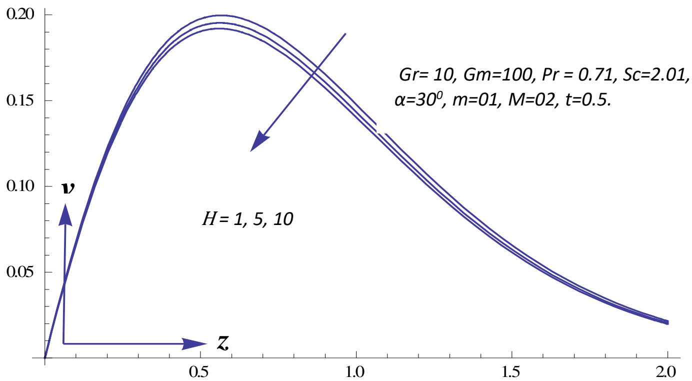

Figure 2.6. Velocity v for different values of *H*.

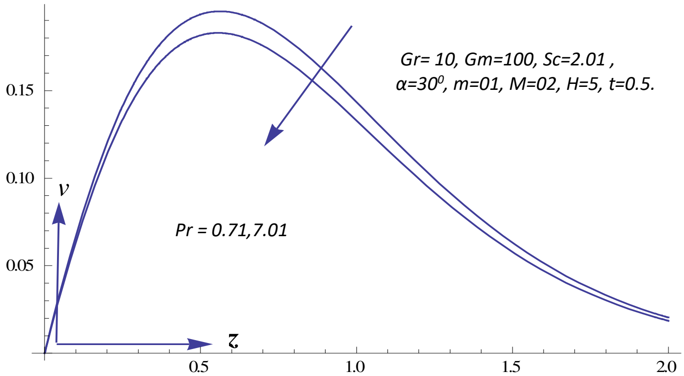

Figure 2.7. Velocity v for different values of *Pr*.

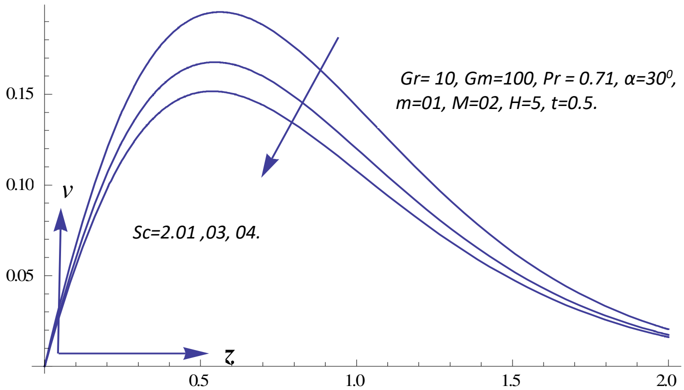

Figure 2.8. Velocity v for different values of *Sc*

Table 1. Skin friction for different parameter (α in degrees)

α	M	m	Pr	Sc	Gm	Gr	H	t	τ_x	τ_y
15	2	1.0	0.71	2.01	100	10	05	0.2	1.1195290	-0.223340
30	2	1.0	0.71	2.01	100	10	05	0.2	0.8472360	-0.175820
60	2	1.0	0.71	2.01	100	10	05	0.2	-0.150420	0.0016993
30	5	1.0	0.71	2.01	100	10	05	0.2	-0.717858	1.0649428
30	2	5.0	0.71	2.01	100	10	05	0.2	1.2885115	0.0212287
30	2	1.0	7.01	2.01	100	10	05	0.2	1.0009976	0.2649130
30	2	1.0	0.71	3.00	100	10	05	0.2	0.0432260	-0.612874

30	2	1.0	0.71	4.00	100	10	05	0.2	0.3934309	-0.188970
30	2	1.0	0.71	2.01	050	10	05	0.2	-0.337800	-0.194743
30	2	1.0	0.71	2.01	100	50	05	0.2	0.8088639	-1.975099
30	2	1.0	0.71	2.01	100	10	10	0.2	1.1059549	0.1179261
30	2	1.0	0.71	2.01	100	10	05	0.4	5.4633980	-0.568607

Table 2. Nusselt number for different parameter

H	*Pr*	*t*	*Nu*
05	0.71	0.4	-2.9533214
10	0.71	0.4	-3.7673379
05	4.00	0.4	-2.2309082
05	7.01	0.4	-2.9512142
05	0.71	0.3	-2.3535000
05	0.71	0.5	-3.5488468

4. Conclusion

In this paper a theoretical analysis has been done to study the effect of heat absorption on unsteady MHD flow past an impulsively started inclined plate with variable wall temperature and mass diffusion in the presence of Hall current. The results obtained for velocity and skin friction are in agreement with the actual flow. It is observed that the primary velocity increases with increasing the values of Hall parameter and heat absorption parameter. The secondary velocity decrease with increasing the values of Hall parameter heat absorption parameter. It is also found that the skin friction decreases with increasing the values of Hall parameter and heat absorption parameter. The Nusselt number decreases with heat absorption parameter.

Appendex:

$$a = \frac{M(1-im)}{1+m^2}.\ A_1 = erfc\left[\frac{-2Hit - z\sqrt{P_r}}{2\sqrt{t}}\right],\ A_2 = erfc\left[\frac{-2Hit + z\sqrt{P_r}}{2\sqrt{t}}\right],$$

$$A_3 = (-1 - A_{17} + e^{2\sqrt{a}z}(A_{18}-1)),\ A_4 = (1 + A_{17} + e^{2\sqrt{a}z}(A_{18}-1)),$$

$$A_5 = (-1 + A_{19} + exp(2z\sqrt{\frac{(a-H)P_r}{Pr-1}})(A_{20}-1)),$$

$$A_6 = (1 + A_{21} + \exp(2\sqrt{a}z)(1 - A_{22})),\ A_7 = (-1 + A_{23} + \exp(2z\sqrt{\frac{aS_c}{S_c - 1}})(A_{24} - 1)),$$

$$A_8 = (1 + A_{21} + \exp(2\sqrt{a}z)(A_{22} - 1)),\ A_9 = (-1 - A_{21} + \exp(2\sqrt{a}z)(A_{22} - 1)),$$

$$A_{10} = (1 + A_{25} + exp(-2Hiz\sqrt{P_r})(A_{26} - 1)), A_{11} = (1 - A_{25} + exp(-2Hiz\sqrt{P_r})(A_{26} - 1)),$$

$$A_{12} = (-1 - A_{27} + exp(2z\sqrt{\frac{(a-H)P_r}{P_r - 1}})(A_{28} - 1)),$$

$$A_{13} = (-1 - A_{29} + \exp(2z\sqrt{\frac{aS_c}{S_c - 1}})(A_{30} - 1)),$$

$$A_{14} = exp(\frac{at}{P_r - 1} - z\sqrt{\frac{(a-H)P_r}{P_r - 1}} - \frac{HtP_r}{P_r - 1}),\ A_{15} = \exp(\frac{at}{S_c - 1} - z\sqrt{\frac{aS_c}{S_c - 1}}),$$

$$A_{16} = (-1 + erf(\frac{z\sqrt{S_c}}{2\sqrt{t}})),\ A_{17} = erf(\sqrt{at} - \frac{z}{2\sqrt{t}}),\ A_{18} = erf(\sqrt{at} + \frac{z}{2\sqrt{t}}),$$

$$A_{19} = erf(\frac{z}{2\sqrt{t}} - \sqrt{\frac{(a-H)tP_r}{P_r - 1}}),\ A_{20} = erf(\frac{z}{2\sqrt{t}} + \sqrt{\frac{(a-H)tP_r}{P_r - 1}}),$$

$$A_{21} = erf(\frac{\sqrt{at} - z}{2\sqrt{t}}),\ A_{22} = erf(\frac{\sqrt{at} + z}{2\sqrt{t}}),\ A_{23} = erf(\frac{z - 2t\sqrt{\frac{aS_c}{S_c - 1}}}{2\sqrt{t}}),$$

$$A_{24} = erf(\frac{z + 2t\sqrt{\frac{aS_c}{S_c - 1}}}{2\sqrt{t}}),\ A_{25} = erf(-Hi\sqrt{t} - \frac{z\sqrt{P_r}}{2\sqrt{t}}),$$

$$A_{26} = erf(-Hi\sqrt{t} + \frac{z\sqrt{P_r}}{2\sqrt{t}}), A_{27} = erf(\sqrt{t}\sqrt{\frac{a-H}{P_r - 1}} - \frac{z\sqrt{P_r}}{2\sqrt{t}}),$$

$$A_{28} = erf\left(\sqrt{\frac{(a-H)t}{P_r - 1}} + \frac{z\sqrt{P_r}}{2\sqrt{t}} \right),\ A_{29} = erf\left(\frac{1}{2\sqrt{t}} \left(\sqrt{\frac{at}{S_c - 1}} - z\sqrt{S_c} \right) \right),$$

$$A_{30} = erf\left(\frac{1}{2\sqrt{t}} \left(\sqrt{\frac{at}{S_c - 1}} + z\sqrt{S_c} \right) \right),$$

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