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The study of the elastic deformation of a flexible wheel by a cam wave generator

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The paper presents the results of research on the study of the elastic deformation of a flexible wheel from a double harmonic transmission, under the action of a cam wave generator. Knowing exactly how the flexible wheel is deformed is important in correctly establishing the geometric parameters of the wheels teeth, allowing a better understanding and appreciation of the specific conditions of harmonic gearings in the two stages of the transmission. The veracity of the results of this theoretical study on the calculation of elastic deformations and displacements of points located on the average fiber of the flexible wheel was subsequently verified and confirmed by numerical simulation of the flexible wheel, in the elastic field, using the finite element method from SolidWorks Simulation.

Keywords: *deformation, displacement, flexible wheel, cam wave generator, double harmonic transmission*

1. Introduction

Toothed harmonic transmissions are widely used in the construction of numerous automation equipment and high-performance systems, such as: cosmic ships and rockets, airplanes, helicopters, radar antennas, joints of industrial robots, atomic reactors, naval mechanisms, servo-mechanisms, motors-reducers, machine tools, shareholders in hermetic spaces, etc. [1-5].

However, the durability of harmonic transmissions is limited, being conditioned by the durability of the flexible wheel, which is subject to complex dynamic stresses, both from the wave generator and from the fixed rigid wheel.

Since the advent of harmonic transmission, many specialists have focused on researching the dynamic behavior of the toothed flexible wheel. Thus, Sahoo and Maiti investigated the state of stresses and strains in the case of a long flexible wheel cup-type deformed by a cam wave generator [6], Zou et al. determined stress and deformation states of short flexible wheel, also of the cup-type deformed by a cam wave generator [7]. Folega chose as a model of analysis a short flexible wheel with external teeth, deformed by a wave generator with discs [8], and Dong et al. they established an analysis model also consisting of the long flexible wheel cup-type, and took into account the action of the alternative dynamic load [9].

In this context, the present paper aimed to investigate the deformation of short flexible wheel from a double harmonic transmission, under the action of a cam wave generator.

Double harmonic transmissions are part of the category of toothed harmonic transmissions, characterized by a high orientation accuracy, compact and coaxial construction, small backlash, low axial size and small mass, high transmission ratio, etc. [10], [11]. The structure of the double harmonic transmission subjected to the research is presented in figure 1. It consists of: cam wave generator (1), flexible wheel (2), fixed rigid wheel (3) and mobile rigid wheel (4).

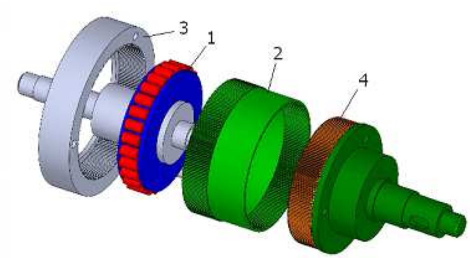

Figure 1. The structure of the double harmonic transmission

In the case of a double harmonic transmission, the short flexible wheel has the shape of a flexible circular tube with a thin wall, which is open at both ends and is provided at each end with teeth (at one end whit outer teeth and at the other end whit inner teeth).

By forcibly mounting the wave generator (1) inside the flexible wheel (2), it deforms elliptically and will have four equidistant engagement zones: two with fixed rigid wheel (3 - having inner teeth) in the left front plane of the flexible wheel and two others with mobile rigid wheel (4 - having outer teeth) in the right front plane of the flexible wheel. There is a 90° angle between the two pairs of opposite gear areas.

2. The deformation of the flexible toothed wheel

The functional performance and durability of the double harmonic transmission are largely influenced by the manufacturing accuracy of the flexible wheel and the wheel deformation law, which in turn depends on its geometry, the type of wave generator and how it is coupled to the output shaft.

In order to investigate the deformation of the flexible wheel, it is admitted the hypothesis that the average fiber of the deformed wheel keeps its elliptical shape and constant length in any cross section of it, because the deformations produced are in the elastic domain.

It has also been admitted that the elliptical deformation of the flexible wheel is performed according to the cosinusoidal law, which can be reproduced with a good approximation, with the help of a generator with two deformation waves, with an elliptical profiled cam (figure 2).

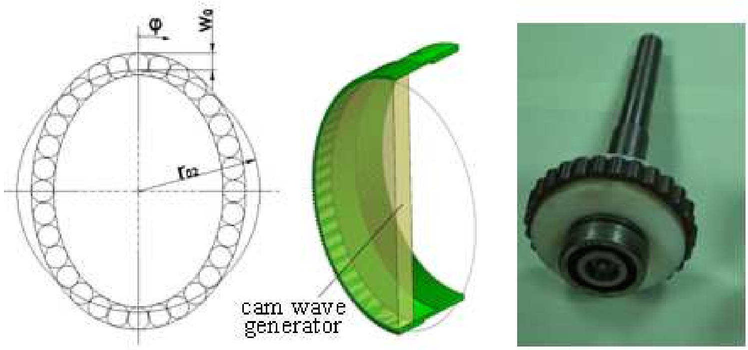

Figure 2. The model and construction of the cam wave generator

The static pictures of the deformation of the short flexible wheel, in its two frontal cross-sections, were defined by means of the calculation relations of the radial (w, w ') and tangential (v, v') displacements of the points located on the average fibers from the two front sections of the wheel as well as the angle of rotation of the normal ((θ, θ'), valid for the case of a cam wave generator:

$$w = w_0 \cdot \cos 2\varphi \qquad (1)$$

$$v = -(w_0 / 2) \cdot \sin 2\varphi \qquad (2)$$

$$\theta = (3 w_0 / 2r_0) \cdot \sin 2\varphi \qquad (3)$$

where: w_0 - is the maximum radial deformation; r_{02} - the average fiber radius of the undeformed flexible wheel; φ - the angular parameter of the cam wave generator.

The calculation relations for the elastic deformations (w, v, θ) of the flexible wheel in the front section where the wave generator operates they are valid and for the front section at the opposite end of the wheel (to the calculation w', v', θ'), with the observation that the maximum radial deformation value w_0 will be replaced with $w_0' = 0.27$ mm.

Figure 3 shows the variation diagrams of the radial displacements (w; w'), tangential (v, v') and of the rotation angle of the normal (θ; θ') depending on the angular parameter φ of the cam wave generator, for points located on the middle fiber in the two front sections of the deformed flexible wheel (characterized by: r_{02} = 29.3 mm; w_0 = 0.3 mm and w_0'= 0.27 mm).

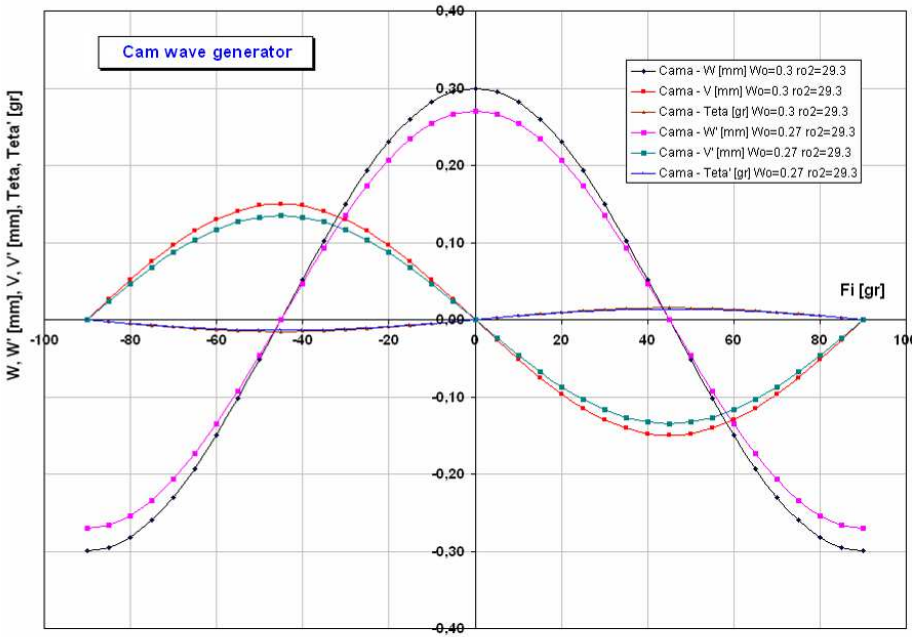

Figure 3. Diagrams of elastic deformations of the short flexible wheel

From the comparative analysis of the curves of variation of radial and tangential displacements it can be seen that these curves keep the same shape in both front sections of the flexible wheel, and the maximum radial deformation in the front section where the cam wave generator operates always has the value greater than the maximum radial deformation in the other front section.

Figure 4 shows the dependence between radial displacements (w, respectively w ') and tangential ones (v, respectively v'), for the case of static deformation of the short flexible wheel by the cam wave generator.

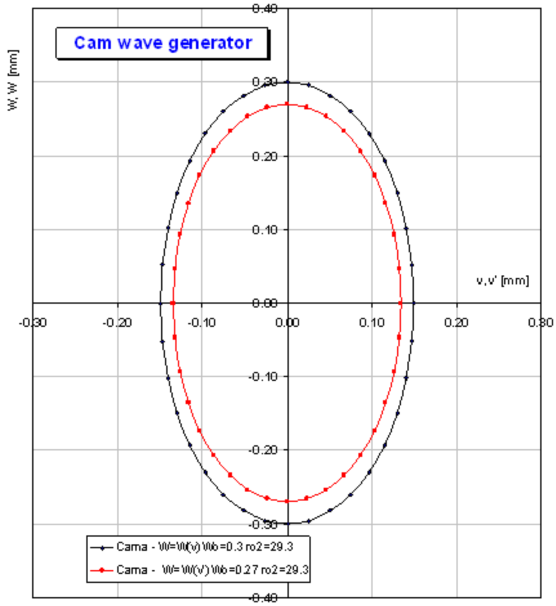

Figure 4. Radial displacement diagram as a function of tangential displacement

3. Numerical simulation of the flexible wheel dynamic

The numerical simulation of the flexible wheel dynamic was performed for the case of elastic deformation of the wheel by a cam wave generator, using the finite element method, from the SolidWorks Simulation module [12].

In the numerical calculation, the flexible wheel was modeled by an open cylinder at both ends, defined by: radius r_{02} = 29.3mm, length l = 30 mm and wall thickness s = 0.6 mm, which is provided at the ends with the teeth (outer and inner, respectively) having a width equal to 12 mm.

Within the numerical analysis performed, the variations of von Mises displacements and tensions in the flexible wheel body were studied, depending on the loading moment of the transmission.

Figure 5 shows, in the form of color maps, the distribution of the von Mises tensions ($\sigma_{von\ Mises}$) and the resulting displacement (Δ) of the flexible wheel body, for a load M = 100 Nm.

It has been found that the stresses arising in the flexible wheel wall are dependent on the transmission load, having a slightly increasing character as the transmission load increases. The maximum tension appears in the area of contact with the wave generator, in the direction of the large axis of the ellipse.

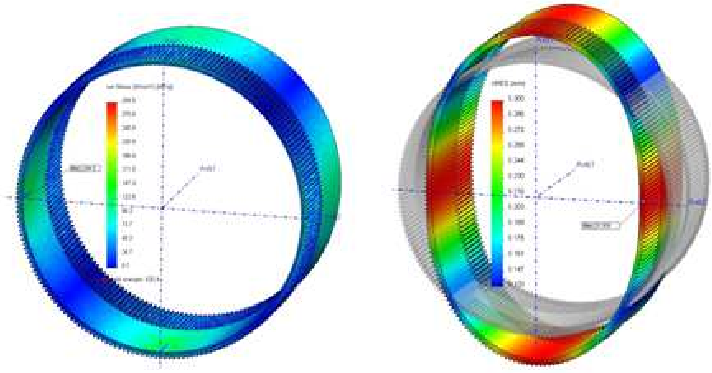

Figure 5. Distribution of $\sigma_{\text{von Mises}}$ tension and resulting displacement Δ

Figure 6 shows, in the form of color maps, the resulting displacements Δ of the characteristic nodes located on the generator N, respectively on the generator E of the deformed flexible wheel, for a load M = 100 Nm.

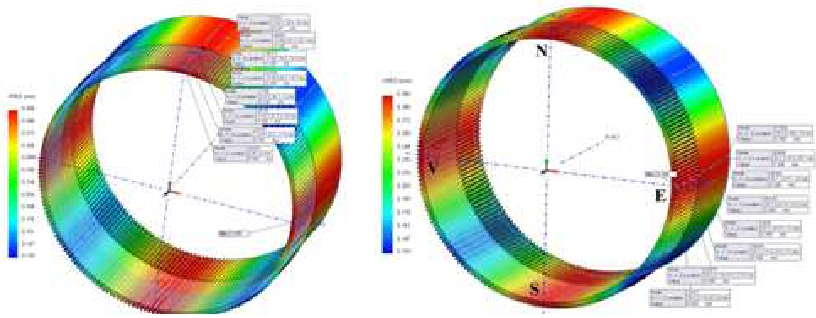

Figure 6. The variation of the displacement Δ along the generators N and E

Figure 7 shows the variation curves of the displacements of the nodes positioned on the two generators (N and E) of the flexible toothed wheel, for a load M = 100 Nm.

From the analysis of the resulting displacement variation diagram Δ = Δ (z), it is observed that in the area of the application point of the deformation force of the flexible wheel (corresponding to the position z = 0, on the generator N) is obtained the value of the maximum radial deformation (Δ (0) = w_0 = 0.3 mm).

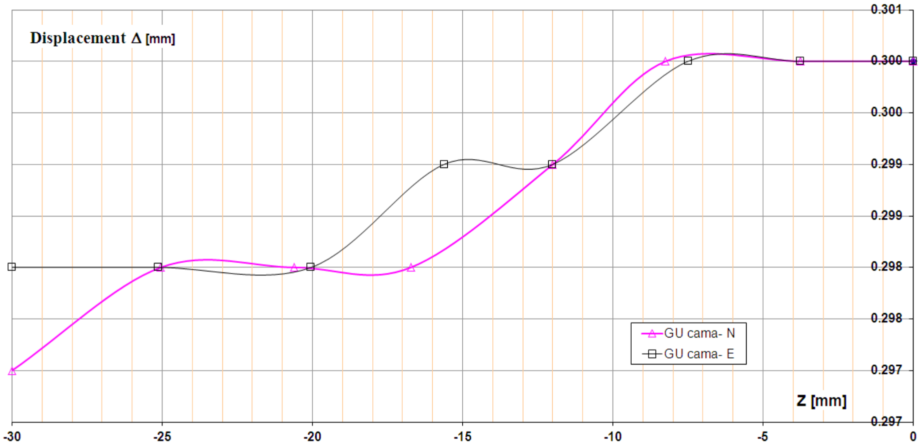

Figure 7. The variation of the displacement Δ along the generators N and E

It is observed that the resulting displacement of the nodes located on the two generators of the deformed flexible wheel has a slightly decreasing character in relation to the increase of the positioning distance of the node with respect to the NSVE plane of the flexible wheel.

4. Conclusion

The paper presents the results obtained following the application of an analytical calculation method of the elastic deformations of the flexible wheel, which is based on a cosinusoidal law of wheel deformation by a cam wave generator.

The correctness of these results was verified and confirmed by numerical simulations performed on the flexible wheel of the double harmonic transmission, using the finite element method from the SolidWorks Simulation program.

From the comparative analysis of these results it was found that the tensions in the flexible wheel wall are dependent on the transmission load and have a slightly increasing character as the load increases. The maximum tension appears in the contact area with the cam wave generator, in the direction of the large axis of the ellipse.

From the study of the variation curves of the resulting displacement of the nodes located on the two generators (N, E) it is observed that the resulting displacement has a slightly decreasing character with the increase of the positioning distance of the nodes in relation to the NSVE face of the flexible wheel, fact confirmed by the results of the analytical study.

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