

## **Contributions to the synthesis and modeling of gear wheels geometry of a double harmonic transmission**

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*The paper presents the geometric synthesis and modeling of the geometry of the three gear wheels (flexible, fixed rigid and mobile rigid) of a double harmonic transmission. The geometric design of the wheels and their teeth was made by using particular relationships that resulted by imposing the specific conditions of harmonic engagement that exist between these wheels. Modeling of the 3D geometry of the wheel gears of the considered double harmonic transmission was performed in the SolidWorks CAD program, based on the geometric parameters resulting from performing the geometric synthesis.*

**Keywords:** *synthesis, modeling, geometry, gear wheel, double harmonic transmission*

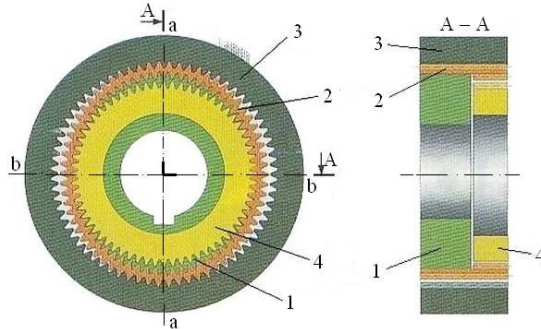
### **1. Introduction**

In the literature, numerous works have highlighted, graphically or analytically, the fact that the gearing conditions between the wheel gears of harmonic transmissions are determined mainly by two factors, namely: teeth geometry and deformation curve shape of the flexible wheels. (which also changes depending on the transmission torque and the existing clearances between the transmission elements) [1-6].

In general, the teeth of the flexible wheels of the toothed harmonic transmissions materialize with small modules, so that the teeth have very small dimensions in relation to the dimensions of the wheel and can be considered non-deformable and embedded in its body, and the wheel deforms [7-9].

Geometric synthesis and modeling was performed for the case of a double harmonic transmission (figure 1) which has as a whole the following four elements:

wave generator with cam (1) - as input element, toothed flexible wheel (2) - as intermediate element, fixed rigid wheel (3) and mobile rigid wheel (4) - as output element [10].



**Figure 1.** Scheme of the double harmonic transmission

In this transmission, the flexible toothed wheel (2) has the shape of a short, thin-walled flexible circular tube, which is open at both ends and is provided at one end with outer teeth ( $z_2$ ) and at the other with inner teeth ( $z'_2$ ).

After assembling the transmission, the cam wave generator (1) is in sliding contact on the entire periphery of the flexible wheel (2), which it deforms elliptically, so that it will have four areas of harmonic engagement, equidistant, two opposite with fixed rigid wheel (first stage of harmonic gear - in direction a-a), respectively two with mobile rigid wheel (second stage of harmonic gear - in direction b-b).

## 2. Geometric synthesis of double harmonic transmission

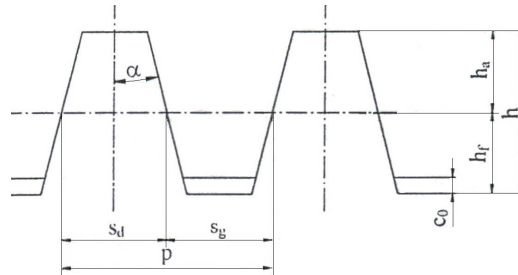
The geometric synthesis of the double harmonic transmission aimed at the dimensional-geometric definition of the wheels and their teeth, so as to ensure the imposed functional conditions.

Namely, that the rotational motion is transmitted, with a constant transmission ratio, by the propagation of some elastic deformations on the periphery of the flexible wheel, in the form of harmonic waves.

From the study of the kinematic conditions of the double harmonic transmission, presented in [11-13], it resulted that the optimal profile of the wheels teeth is the rectilinear one, and the teeth in cross section will have a triangular shape (figure 2).

Within the geometric synthesis of the double harmonic transmission with 2-wave deformation generator and with the rectilinear profile of the wheel teeth, the characteristic geometric elements of the wheels and their teeth were determined

according to the tooth modulus ( $m$ ), by using particular relations which are shown in Table 1.



**Figure 2.** The rectilinear profile of the teeth

**Table 1.** Geometric synthesis algorithm

Geometric parameter	Calculation relationship
Tooth pitch	$p = \pi \cdot m$
Addendum	$h_a = 7/8 \cdot m$
Dedendum	$h_f = 9/8 \cdot m$
Tooth height/total depth	$h = 2 \cdot m$
Maximum radial clearance	$c_0 = 0,3 \cdot m$
Tooth thickness on the pitch circle	$s_d = 7/16 \cdot \pi m$
Width of the space on the pitch circle	$s_g = 9/16 \cdot \pi m$
Half-angle of the tooth profile	$\alpha = \text{arctg } \pi / 5,76$
Half-angle of the space between the teeth	$\alpha + \Delta\alpha = \alpha + \text{arctg } (3,6693 \cdot m/d_3)$ $d_3$ - pitch diameter of fixed wheel
Tooth width/face width	$b = (0,1 \dots 0,3) \cdot d_i$ ; $d_i$ - inner diameter of flexible wheel
Pitch diameter	$d = m \cdot z$
Outside diameter	$d_a = d \pm 2h_a$
Inside diameter	$d_f = d \mp 2h_f$
Wall thickness of flexible wheel in the toothless area	$s = (0,01 \dots 0,02) \cdot d_i$
Length of flexible wheel	$l = s / (0,02 \dots 0,03)$

In table 1, in relations where two signs appear simultaneously, the upper sign for the outer gear wheel is accepted, respectively the lower sign for the inner gear wheel.

Following the application of the proposed algorithm, in the case of the double harmonic transmission with a transmission ratio = 48.47, the values given in table 2 and table 3 were obtained for the main geometric parameters of the gear wheels.

**Table 2.** Geometric parameters of the gear wheels in the first engagement

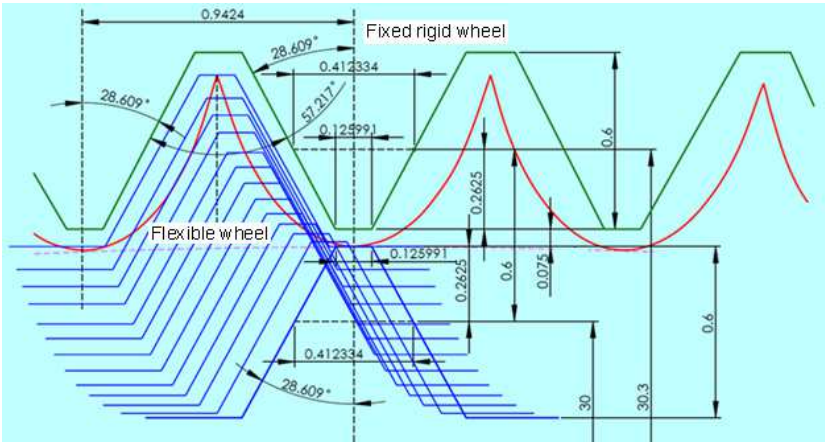
Geometric parameter	Value
Teeth module	$m = 0.3 \text{ mm}$
Tooth pitch	$p = 0.9424 \text{ mm}$
Maximum radial deformation	$w_0 = 0.3 \text{ mm}$
Number of teeth of the wheel: fixed/flexible	$z_3 = 202 / z_2 = 200$
Difference in wheel tooth numbers	$n_u = z_3 - z_2 = 2 \text{ dinți}$
Pitch diameter of the wheel: fixed/flexible	$d_3 = 60.6 \text{ mm} / d_2 = 60 \text{ mm}$
Outside diameter of the wheel: fixed/flexible	$d_{a3} = 60.075 \text{ mm} / d_{a2} = 60.525 \text{ mm}$
Inside diameter of the wheel: fixed/flexible	$d_{f3} = 61.275 \text{ mm} / d_{f2} = 59.325 \text{ mm}$
Inner diameter of the flexible wheel	$d = 58.125 \text{ mm}$
Total depth	$h = 0.6 \text{ mm}$
Addendum	$h_a = 0.2625 \text{ mm}$
Dedendum	$h_f = 0.3375 \text{ mm}$
Maximum radial clearance	$c_0 = 0.06 \text{ mm}$
Half-angle of the tooth profile	$\alpha_2 = 28^\circ 36' 31''$
Half-angle of the space between the teeth	$\alpha_2 + \Delta\alpha_2 = 29^\circ 38' 51''$
Tooth thickness on the pitch circle	$s_d = 0.4123 \text{ mm}$
Width of the space on the pitch circle	$s_g = 0.5301 \text{ mm}$
Tooth width of the wheel: fixed/flexible	$b_3 = 12 \text{ mm}; b_2 = 12 \text{ mm}$
Length of the wheel: fixed/flexible	$l_3 = 16 \text{ mm}; l_2 = 30 \text{ mm}$
Wall thickness of flexible wheel	$s = 0,6 \text{ mm}$

**Table 3.** Geometric parameters of the gear wheels in the 2<sup>nd</sup> engagement

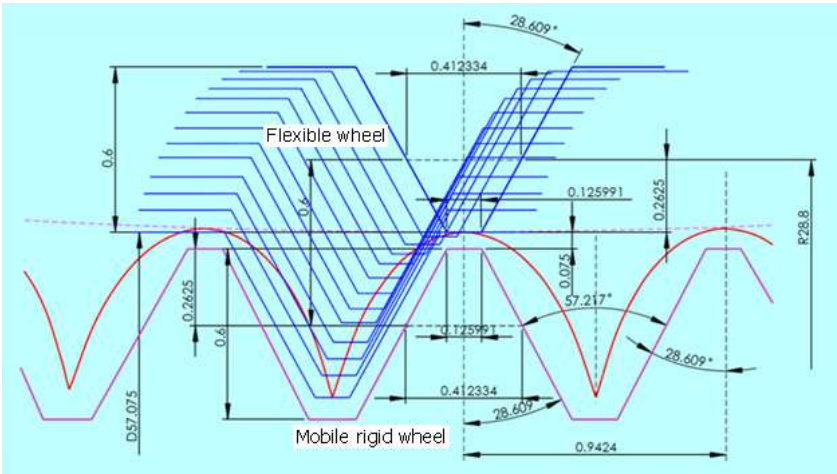
Geometric parameter	Value
Teeth module	$m = 0.3 \text{ mm}$
Tooth pitch	$p = 0.9424 \text{ mm}$
Maximum radial deformation	$w'_0 = 0.27 \text{ mm}$
Number teeth of the wheel: flexible/mobile	$z'_2 = 192 / z_4 = 190$
Difference in wheel tooth numbers	$n_u = z'_2 - z_4 = 2 \text{ dinți}$
Pitch diameter of the wheel: flexible/mobile	$d'_2 = 57.6 \text{ mm} / d_4 = 57 \text{ mm}$
Outside diameter of wheel: flexible/mobile	$d'_{a2} = 57.075 \text{ mm} / d_{a4} = 57.525 \text{ mm}$
Inside diameter of the wheel: flexible/mobile	$d'_{f2} = 58.275 \text{ mm} / d_{f4} = 56.325 \text{ mm}$
Total depth	$h = 0.6 \text{ mm}$
Addendum	$h_a = 0.2625 \text{ mm}$
Dedendum	$h_f = 0.3375 \text{ mm}$
Tooth thickness on the pitch circle	$s_d = 0.4123 \text{ mm}$
Width of the space on the pitch circle	$s_g = 0.5301 \text{ mm}$
Tooth width of the wheel: flexible/mobile	$b'_2 = 12 \text{ mm}; b_4 = 12 \text{ mm}$
Length of the wheel: flexible/mobile	$l_2 = 30 \text{ mm}; l_4 = 12 \text{ mm}$

The demonstration of the veracity of the values of the geometric parameters of the gears of the double harmonic transmission was made by confirming the fulfillment of the conditions of existence of the harmonic gears in the 2 stages of the transmission.

Figure 3 shows the image of the harmonic engagement in the first stage of the transmission, and in figure 4 shows the image of the harmonic engagement in the second stage.

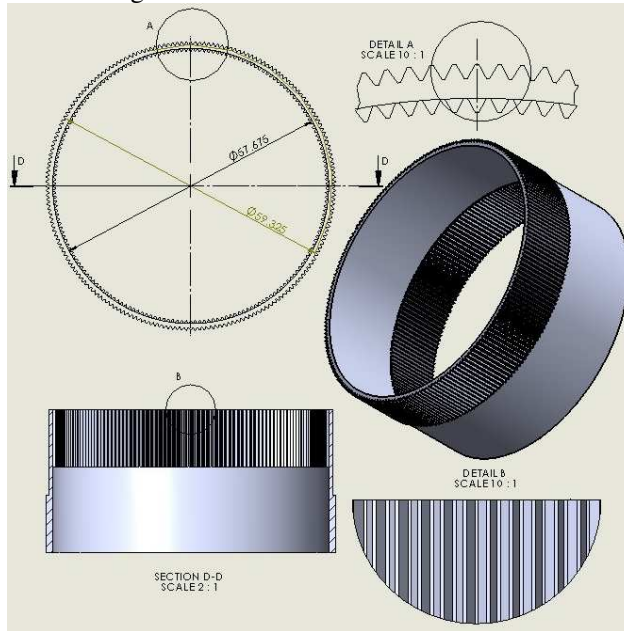


**Figure 3.** The image of the harmonic engagement in the first stage



**Figure 4.** The image of the harmonic engagement in the second stage

The constructive shape and the geometries of the two toothings of the flexible wheel are presented in figure 5.



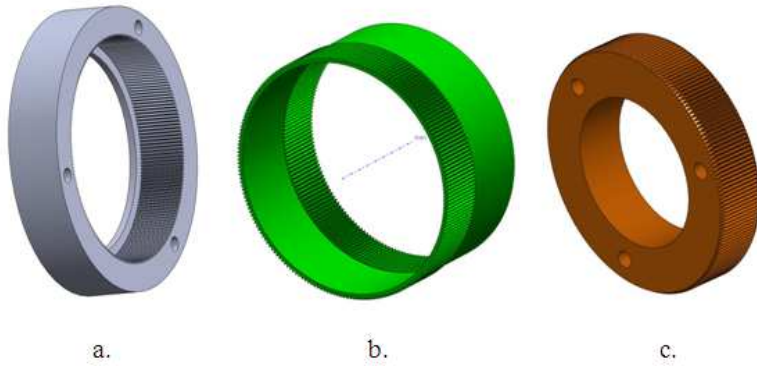
**Figure 5.** The geometry of the flexible wheel

### **3. Modeling the geometry of the gear wheels**

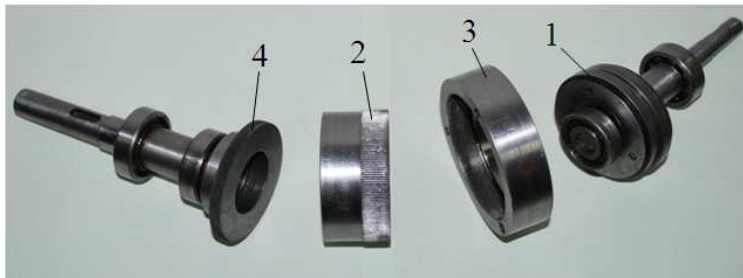
The CAD module of the SolidWorks design program was used to generate the models of the three gear wheels of the double harmonic transmission (fixed rigid wheel, flexible wheel and mobile rigid wheel).

The modeling of the 3D geometry of the three gear wheels was done, in principle, by going through the same stages, namely: elaboration of the annular section and its extrusion, extrusion of the minimum diameter, extrusion of the tooth profile and its circular multiplication [14].

The geometric models of the three gear wheels of the double harmonic transmission are presented in figure 6, a – for the fixed rigid wheel, b – for the flexible wheel, c – for the mobile rigid wheel, and in figure 7 the physical constructions of these elements.



**Figure 6.** The geometric models of gear wheels



**Figure 7.** Construction of double harmonic transmission elements

#### 4. Conclusion

The paper presents and proposes a unitary method of geometric synthesis of double harmonic transmissions, which aimed to determine the relations of defining the characteristic geometric parameters of the gear wheels that are part of the respective transmissions.

Based on the presented calculation relations and the computer-aided design, the basic parameters that characterize the harmonic gears from the 2 stages of the transmission were determined, namely: the tooth profile angle ( $\alpha$ ), the tooth height ( $h$ ) and the maximum radial deformation size of the flexible wheel ( $w_0, w'_0$ ).

The geometric models generated for the component gear wheels (fixed rigid, flexible and mobile rigid) were the basis for their physical realization and for the constitution of the double harmonic transmission assembly.

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