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Improvement of Jain's algorithm for frequency estimation

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In this paper we propose a procedure to correct Jain's algorithm, which in certain situations fails in correctly estimating the frequency by indicating frequency values that are very far from the real frequency. It happens because the two points considered for the method proposed by Jain are not on the same lobe. Thus, a method is proposed according to which these points are chosen so that the results are improved.

Keywords*: frequency estimation, interpolation, algorithm, Discrete Fourier Transform, modal analysis*

1. Introduction

In numerous engineering applications involving measurements to estimate the frequencies of signals, such as damage detection [1], telecommunication [2], radar/sonar [3] it is crucial to accurately estimate the frequencies. Usually, the frequencies are estimated involving the Discrete Fourier Transform (DFT), but the results depend rather on the sampling strategy as on the frequency of the signal. In order to improve frequency readability, interpolation methods are used, with the help of which the maximum of the interpolation curve is determined. So, the frequency is given by the inter-line position of the maximum of the curve. These interpolation methods take into account the largest sample magnitude from DFT and one or two neighbors. Some methods use polynomial functions for interpolation [4,5], others an asymmetric *pseudo-sinc* function [6,7].

The literature presents interpolation methods based on different algorithms that use two or three points from the module spectrum [8]-[11] or the complex values of the spectrum [4,5,12]. When the accuracy of these interpolation methods is analyzed [6,13,14], it was found that in some of them, when the signal contains an integer number of cycles, the correct frequency is obtained [10,11], but there are also methods that introduce big errors in that case [9]. Among the latter methods we choose that developed by Jain, and here we present how it can be improved by selecting the proper points for interpolation. We show, by comparing the results obtained with the original and improved algorithm, the superiority of the latter.

2. Jain's algorithm for frequency estimation

Let us consider a sinusoidal signal as a sampled sequence

$$a[n] = A\exp\left[j2\pi \frac{f_0}{f_s} n \right],\ n = 0,...,N-1 \tag{1}$$

where A is the amplitude, f_0 is the frequency of the signal, f_S is the sampling rate, N the number of acquired samples, n the index of the samples and $j^2 = -1$. From the sampling strategy, we obtain the distance between two spectral lines, i.e. the is the frequency resolution, that is:

$$\Delta f = \frac{f_s}{N} \tag{2}$$

The discrete Fourier transform DFT of $a[n]$ at spectral line k displays the amplitude:

$$A_k = \sum_{n=0}^{N-1} a[n] e^{-j(2\pi/N)nk} \tag{3}$$

In order to determine the true frequency at an inter-line position, Jain's algorithm involves two stages. In the first stage an index k of the largest magnitude samples $|A_k|$ from DFT is determined and in the second stage the target is finding a correction coefficient δ in the vicinity of k and determine the inter-bin frequency location (the corrected frequency) with the formula:

$$f_c = (k+\delta)\Delta f \tag{4}$$

To find the correction coefficient [11], Jain uses two points from the spectrum of modules. The method developed by Jain is based on the following algorithm:

1. Select the points for interpolation and calculate the amplitude ratio:
   if $|A_{k-1}| > |A_{k+1}|$, then $\alpha_1 = |A_k| / |A_{k-1}|$
   else, $\alpha_2 = |A_{k+1}| / |A_k|$
2. Calculate the correction term:
   if α_1 is selected, than $\delta_1 = \alpha_1 / (1+\alpha_1)$
   else, $\delta_2 = \alpha_2 / (1+\alpha_2)$
3. Calculate the corrected frequency:

for δ_1 use the relation

$$f_c = (k - 1 + \delta_1)\Delta f \tag{5}$$

else, use the relation

$$f_c = (k + \delta_2)\Delta f \tag{6}$$

From numerous simulation we found out that the results obtained by involving this algorithm provides good results, except some signal time lengths for which the signal contains almost an entire number of cycles. In next section we show why the algoritm fails for these time lenghts and how it can be improved.

3. Correction of Jain's algorithm

Thus, in the proposed algorithm, namely the corrected Jain's algorithm, the two points selected for interpolation must belong to the main lobe, even if the point that is the neighbour of the maximizer has a smaller amplitude as the second neighbour. In [17], an algorithm for selecting the two points with which we will correct Jain's algorithm is proposed. It involves following steps:

- Add N_{ZP}=2 points with zero amplitude to the original signal, it results a zero-padded signal
- Calculate the DFT for the zero-padded signal and determine the coordinates for the maximizer and its neighbors
- Subtract N_{ZP} points from the zero padded signal to obtain the original signal
- Calculate the DFT for the original signal and find the coordinates of the maximizer and its neighbors
- Select the proper neighbor
    - o if $\left|A_k^*\right| < \left|A_k\right|$ (where A_k^* the amplitude of the maximizer for the zero-padded signal) then the point with the amplitude $\left|A_k\right|$ is on the left side of the main lobe, the point with the amplitude $\left|A_{k-1}\right|$ is on a secondary lobe and the one with the amplitude $\left|A_{k+1}\right|$ is on the same lobe as the one with the amplitude $\left|A_k\right|$, so we will take for the algorithm the points having the amplitudes A_k and A_{k+1}
    - o else the points with amplitudes $\left|A_{k-1}\right|$ and $\left|A_k\right|$ are on the same lobe and we will take these points for the algorithm

Figure 1 shows such a case in which we will have to correct the algorithm proposed by Jain and choose the two points differently. This is the case where $\left|A_{k-1}\right| < \left|A_{k+1}\right|$ and so according to Jain's algorithm we should calculate α_2 based on the amplitudes $|A_{k+1}|$ and $|A_k|$, then δ_2 and evaluate the frequency with formula (6).

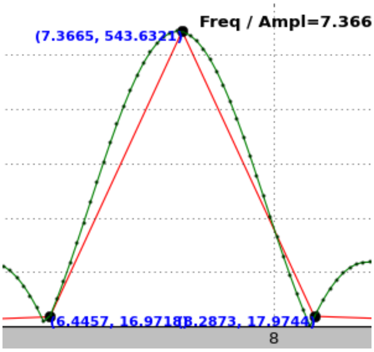

Figure 1. The three points in the DFT spectrum used for interpolation

However, it can be seen that the point having the amplitude $|A_{k+1}|$ is on a secondary lobe and thus we will consider the two points on the same lobe, those having the amplitudes $|A_k|$ and $|A_{k-1}|$, thus calculating α_1, δ_1 and the frequency based on formula (5).

4. Results and discussion

To illustrate the above, we have chosen a signal generated with the given frequency of f_{true} =7.33 Hz and amplitude A_{true} = 1, with a sampling rate f_S = 1000 samples/second. Over time interval of 0.3 s, between t_i = 0.93s and t_f = 1.23s we generated the signal. Thus, starting from 931 samples, adding 5 samples at a time, we finally obtained the signal generated with 1231 samples.

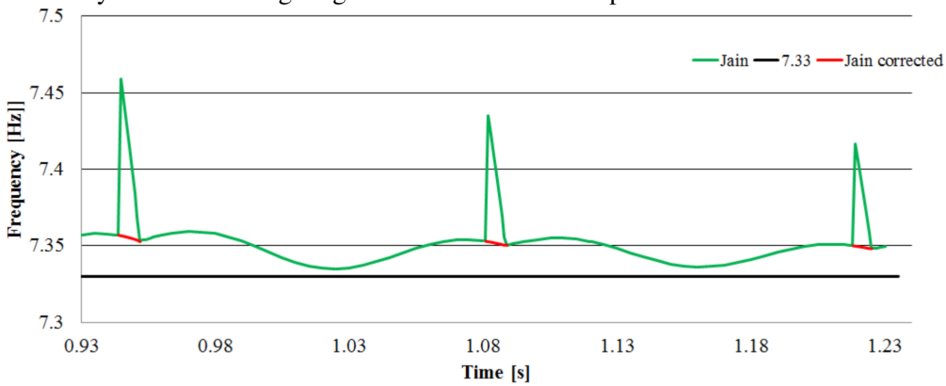

Figure 2. Frequencies estimated with Jain's algorithm and the corrected algorithm

Following the algorithm developed by Jain, we represented graphically (with green line in Figure 2) the estimated frequency using formulas (5) and (6). As we can see, in the area where we have an entire number of cycles this curve suffers a significant jump.

Starting first with N=946 samples and ending with N=953 samples (Figure 3), then from N=1083 samples to N=1089 samples (Figure 4) and from N=1220 samples to N=1225 samples (Figure 5), we chose differently the two points according to the correction algorithm proposed by Jain and presented in paragraph 2, and we obtained the curve with red line. A zoom on the area of interest is presented in figure 3 for 7 entire cycles, figure 4 for 8 cycles and figure 6 for 9 entire cycles.

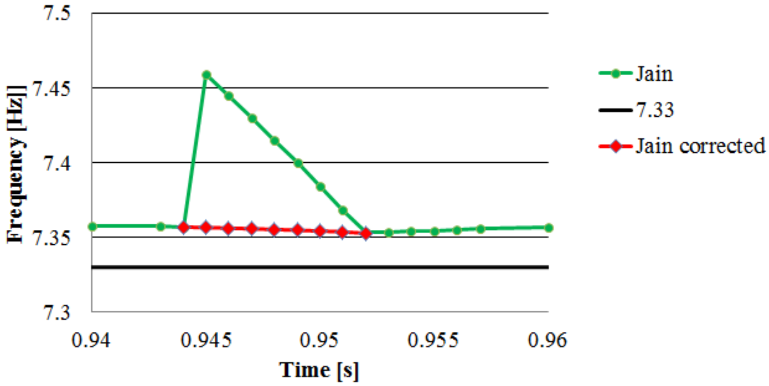

Figure 3. Frequencies estimated with the corrected Jain's algorithm (signal time length around 7 entire cycles)

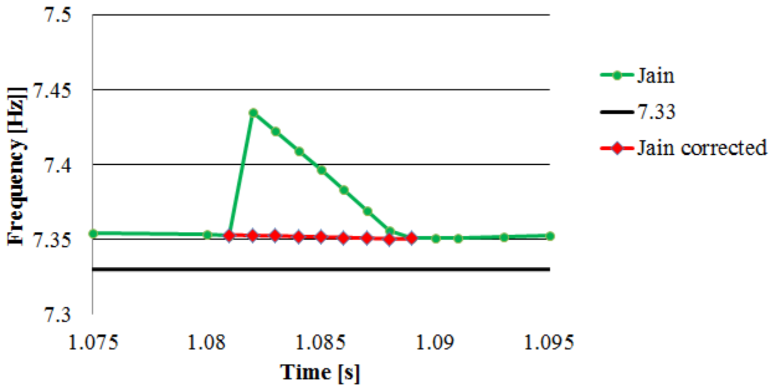

Figure 4. Frequencies estimated with the corrected Jain's algorithm (signal time length around 8 entire cycles)

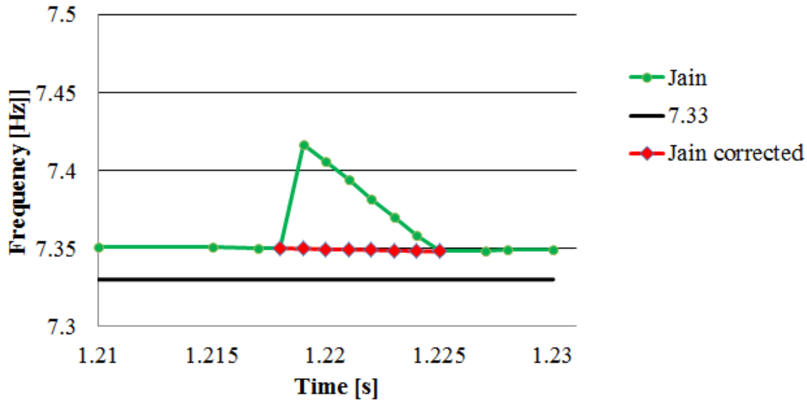

Figure 5. Frequencies estimated with the corrected Jain's algorithm (signal time length around 9 entire cycles)

From figures 3 to 5 one can observe that the error in estimating the frequencies decrease with increasing the time length of the analysed signal for the entire time length range. The improvement of the frequency readability with the corrected Jain's algorithm is obvious.

Table 1. Frequency estimation with Jain's algorithm and the corrected algorithm

T [s]	1.082	1.083	1.084	1.085	1.086	1.087	1.088
N	1083	1084	1085	1086	1087	1088	1089
A_{k-1}	25.21	21.113	17.078	13.103	9.188	5.333	1.541
A_k	541.88	542.765	543.540	544.208	544.769	545.225	545.57
A_{k+1}	25.521	21.748	17.894	13.960	9.948	5.861	1.704
f_{Jain} [Hz]	7.4352	7.42246	7.40947	7.39632	7.38299	7.36949	7.3558
ε_{Jain} [%]	1.437	1.261	1.084	0.905	0.723	0.539	0.352
f_{corr} [Hz]	7.3526	7.35231	7.35197	7.3516	7.35120	7.35079	7.35035
ε_{corr} [%]	0.308	0.304	0.300	0.295	0.289	0.284	0.278

The data used to plot figure 3 is given in table 1, where a special focus concerns the achieved errors. These are denoted as following: (*) the error obtained in estimating the frequency according to the method proposed by Jain is noted with ε_{Jain} ; (**) the error obtained in estimating the frequency according to the proposed correction method is denoted as ε_{corr}. One can observe in table 1 that the proposed correction algorithm makes the maximum error to be reduced from 1.43% as obtained by Jain to 0.3%.

5. Conclusion

The algorithm proposed by Jain that allows to improve frequency readability has been corrected in this paper for different time intervals in which an entire number of cycles occur. This improvement was made by selecting the points to be considered for interpolation so that they are on the same lobe.

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